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External Reputation and Productivity  
In Organizations

*Amiya K. Basu*  
*Lauranne Buchanan*



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University of Illinois Urbana-Champaign



# BEBR

FACULTY WORKING PAPER NO. 90-1675

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University of Illinois at Urbana-Champaign

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External Reputation and Productivity in Organizations

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## ABSTRACT

This paper considers a system consisting of a firm and an agent, both assumed to be risk neutral, where the agent is trying to complete a project in a single time period. The agent selects the amount of effort  $t$  and the level of project difficulty  $a$ , and the firm cannot observe  $t$  or  $a$  directly. The probability of completing the project successfully increases as  $t$  is increased but decreases as  $a$  is increased. If there is a successful completion, which we call a *breakthrough*, the quality of the output equals  $a$ .

The system receives benefits from the external environment if the project is completed successfully. The benefits consist of a fixed reward  $F$  to the firm, a fixed reward  $G$  to the agent, a quality dependent reward  $a$  to the firm, and a quality dependent reward  $Qa$  to the agent.  $Qa$  is the net present value of the reputation the agent will get from the quality achieved by the project.  $Q$  is called the *reputation parameter* and is assumed to be nonnegative.

The agent's compensation from the firm consists of a base salary received regardless of whether a breakthrough occurs, and a reward received upon a breakthrough. It is assumed that the firm is either *naive* or *sophisticated*. The naive firm cannot relate the agent's reward to the quality of the output and can only offer a fixed fee for a breakthrough. The sophisticated firm can base the reward on a perfect measurement of quality and offers the agent a reward of the form  $(s + qa)$ . A comparison of the profitability of the sophisticated firm with that of the naive firm yields the value of the firm's ability to measure the quality of the agent's output.

The paper analyzes how the naive and the sophisticated firm determine optimal compensation plans, and how the results depend on parameters of the environment such as  $F$ ,  $G$  and  $Q$ . Among other results, the paper establishes that if the total fixed reward to the system,  $(F + G)$ , increases, the firm tries to enhance the probability of breakthrough by selecting a compensation plan to induce a lower  $a$ , and when  $(F + G)$  is high enough, even a sophisticated firm selects a compensation plan which induces the agent to choose the lowest level of difficulty,  $a = 0$ . Conversely, an increase in  $Q$  usually leads to the selection of higher levels of difficulty. It is also shown that when  $Q$  is large, the external reputation serves as a sufficient motivator for the agent, and the naive firm may perform reasonably well compared to the sophisticated firm, and may even be exactly as profitable. When  $(F + G)$  is large, both the naive and the sophisticated firms construct compensation plans to induce  $a = 0$ , and once again the naive firm may be as profitable as the sophisticated firm.



## 1. Introduction.

Incentive plans used by organizations have long attracted the attention of economists and researchers in business management. The problem of developing the optimal compensation plan has been studied extensively by scholars in economics (e.g., Holmstrom 1979, Grossman and Hart, 1983) and the related body of research is commonly referred to as *agency theory*. Agency theory has been applied by researchers to different areas of business management such as marketing (e.g., Basu, Lal, Srinivasan and Staelin, 1985), accounting (e.g., Demski and Feltham, 1978), finance (e.g., Fama, 1980) and organizational behavior (e.g., Eisenhardt, 1985). In its simplest form, agency theory considers a dyad consisting of a firm, called a *principal*, and an agent. The productivity of the firm depends on the actions of the agent which the principal cannot observe directly. The principal and the agent are assumed to engage in a leader-follower game where the principal declares a compensation plan based simply on the outcome of the agent's actions which is observable to the principal. The agent responds by selecting the level of effort to expend on behalf of the principal, and his decision depends on his utility for the compensation offered by the principal and his disutility for effort. The principal, in developing the optimal compensation plan, takes the agent's response into account.

In this paper, we analyze a system consisting of a firm and an agent where the agent is trying to complete one project in a given time period. Similar to agency theory, the firm and the agent are assumed to engage in a leader-follower game where the firm declares a compensation plan based on output, and the agent responds by selecting his actions. In contrast with the traditional paradigm, we assume that the actions of the agent have two components: the level of effort the agent expends and the level of project difficulty he attempts. For a given level of difficulty, increased effort enhances the chance of completing the project successfully, an event we call a *breakthrough* throughout the paper. For a given level of effort, the probability of breakthrough is lower for a more difficult project. If a breakthrough is achieved, a more difficult project is recognized for its quality. The level of effort, however, does not affect the quality of the output.

If there is a breakthrough, the firm will always observe it and reward the agent on the basis of the breakthrough. In addition to the firm, the outcome of the agent's efforts is also observed by economic agents outside the system; throughout the paper we refer to these external agents as *external environment* or simply *environment*. The firm and the agent, collectively called the *system*, receives benefits which depend on the quality of the output. In other words, if a breakthrough occurs, the environment rewards the system on the basis of the difficulty of the project undertaken by the agent.

Consider, for example, a university professor about to undertake a research project. The professor recognizes that if he selects a more challenging problem, the probability of suc-

cessfully completing the project declines. If he fails to complete the project, he is unlikely to obtain any tangible output such as a publication. Hence, the professor has the incentive to select problems that are easily solved. The professor, however, also recognizes that solving a more difficult problem is more likely to earn the respect of his peers and further his reputation in his field, and may bring prestigious awards as well as monetary benefits such as research grants and employment offers from other universities. The decision the professor faces is then whether to attempt a smaller project which increases the likelihood of completion, or to concentrate on more difficult problems which have the potential to elevate his stature in the field.

The agent's decision not only affects his own welfare but is also relevant to the employing firm. Part of the grants that come to the professor are appropriated by the university to help support the infrastructure which in turn makes it possible for the professor to do his work. In addition, the reputation of the university, derived in large part from the reputation of its academic staff, makes it easier to acquire other resources that are crucial for the university's survival. A "good" reputation makes it easier to secure funding from state legislatures and grants from the federal government and private companies. It increases the likelihood that the university can continue to attract good students and new faculty which will ensure continued success of the university. It is interesting to note that the firm and the agent may both benefit from the rewards of the environment, and that these benefits may have different origins. The professor, for example, may receive awards from learned societies while the university receives state funding.

The preceding example shows that it is in the interest of the firm to design an incentive plan which relates rewards to the quality of the output. The problem is that the firm may not be sophisticated enough to immediately evaluate the quality of a completed project, thereby making it impossible to reward the agent on this basis. If so, the firm may be forced to use simplifying heuristics in designing the compensation plan. Throughout the paper, we refer to a firm that is capable of compensating an agent on the basis of quality as a *sophisticated firm*, and one that is not able to as a *naive firm*. In the case of the university, for example, the specialization of knowledge may make it difficult for administrators to evaluate the professor's work. As a result, they may rely on the number of publications as a signal for salary and even tenure decisions. Such a reward system appears to be an incentive for the professor to select a large number of simple problems rather than attempt difficult and inherently more risky projects. However, since the agent is motivated both by the firm's compensation plan and the possibility of rewards from the environment, he may still decide to pursue more difficult projects. In addition, the agent may also be driven by the need for internal satisfaction which comes from attaining higher quality. Since this has a similar effect on the agent as external reputation, we do not consider this factor

separately. The important point is that external reputation may help to explain why firms can continue to exist with such seemingly inefficient incentive schemes : in the presence of a sophisticated environment which can evaluate and reward quality, the firm itself has less need to be sophisticated.

While our example focuses on the university, the reader can clearly see how the problem applies to other, commercial settings. Researchers in the R & D division of a commercial firm have to make very similar decisions to the professor discussed above. Industrial salespeople have to decide whether to maintain established accounts or to pursue new and larger accounts. Again, the problems and rewards parallel those of the research setting. The larger account will be more difficult to gain given the competition from other salespeople; however, if acquired, it can bring greater rewards to the agent and the firm in the long run.

The quality dependent reward the professor receives from the environment upon a breakthrough can appropriately be called reputation. In other cases, the nature of the reward may vary. For example, if a researcher develops a commercial product, the reward may be purely monetary. Throughout the paper, we use the term *reputation* to denote the quality dependent reward the agent receives from the environment upon a breakthrough and assume that it can be expressed along a single dimension : its monetary equivalent. In addition, we recognize that the firm and the agent may sometimes receive rewards from the environment which do not depend on quality of the output. A private company which develops a product for the government may receive a fixed reward just for successfully completing the project. The system is likely to receive fixed rewards in the short run before the environment is able to evaluate the quality of the output.

In this paper, we attempt to quantify the effect of the external reputation an agent receives from the environment. The research primarily focuses on how variations in the reputation factor affects the profitability of the firm and the quality the agent tries to achieve. The influence of the reputation factor on the relative performance of a *naive* firm, which cannot reward an agent based on the quality of a completed project, and a *sophisticated* firm, which can, is examined.

The proposed model is distinct from a very important stream of current research in economics and business management which deals with reputation (e.g., Kreps and Wilson, 1982; Milgrom and Roberts, 1982; Datar, 1987; and Cohen, 1988). These works use a sequential game framework to analyze how an economic entity will modify its actions in earlier periods to develop a *reputation* which will be of benefit in later periods. Cohen (1988), for example, uses the example of a lawyer who works hard even if he is not compensated because of the reputation derived for winning. In comparison to this literature, we consider a relationship which exists for one period, even though the results might have long term impact. In addition, in our model reputation depends on a parameter which is introduced exogenously.

The importance of the model is that it recognizes the multidimensional nature of the agent's choices by incorporating the level of difficulty in addition to sheer effort. The simplicity of the framework allows us to obtain several interesting results.

## 2. The Model And Study Objectives.

We first list the assumptions of the model.

### A. The agent's actions.

(a) The agent has two decision variables, the amount of effort to devote,  $t \geq 0$ , and the level of difficulty to attempt,  $a \geq 0$ . The firm cannot observe  $t$  or  $a$  directly.

**B. The probability of breakthrough.** The probability of breakthrough depends on  $t$  and  $a$ , and we denote it by  $p(t, a)$  throughout this paper. We make the following assumptions about  $p(t, a)$ :

(b)  $p(t, a)$  can be expressed as  $p_1(t)p_2(a)$ ,  $p_1(t) \geq 0$ ,  $p_2(a) \geq 0$ .

$p_1(t)$  is a twice differentiable, strictly increasing, and strictly concave function of  $t$ , and  $p_1(0) = 0$ .  $p_2(a)$  is a twice differentiable, strictly decreasing function of  $a$ .

Even though (b) is a simplifying assumption, it has the desirable property that if  $a$  is higher,  $t$  must also be higher in order to have the same chance of breakthrough. Also, an increase in  $a$  reduces the marginal effect of an increase in  $t$  on  $p(t, a)$ .

We make the following three technical assumptions about  $p(t, a)$  :

(c)  $p_1(t)$  is bounded above.

(d)  $\lim_{a \rightarrow \infty} ap_2(a) = 0$ .

(e) If  $a > 0$ ,  $2(p'_2)^2 > p_2 p''_2$ .

**C. System Benefits.** We make the following assumptions about the benefits the system consisting of the firm and the agent may receive from the external environment.

(f) If there is no breakthrough, neither the firm nor the agent receives any benefit from the environment.

If there is a breakthrough, the quality of the output is likely to increase with  $a$ . Also, the rewards of the firm and the agent should increase with quality. For simplicity, we make the following assumptions.

(g) If the agent selects level of difficulty  $a$  and achieves a breakthrough, the quality of the output is  $a$ .

(h) If the agent selects level of difficulty  $a$  and achieves a breakthrough, the monetary equivalent of the benefit the firm receives from the environment has the net present value  $(F + a)$ , and the monetary equivalent of the benefit the agent receives from the environment has the net present value  $(G + Qa)$ .  $F$ ,  $G$  and  $Q$  are constants, and  $Q \geq 0$ .

Thus, the long term benefit of the system from a breakthrough which depends entirely on quality has the net present value  $(1 + Q)a$  of which the firm receives  $a$  and the agent  $Qa$ .  $Qa$  is the monetary equivalent of the reputation of the agent, and throughout this paper we call  $Q$  the *reputation parameter*. If  $Q$  is small, the benefit from quality goes mostly to the firm, while  $Q \rightarrow \infty$  means that this benefit goes primarily to the agent. It should be noted that we do not allow  $Q < 0$  which would indicate that the firm and the agent have incompatible goals.

$F$  and  $G$  are fixed rewards which do not depend on quality. If the environment is fully sophisticated, i.e., it can evaluate quality of output as soon as a breakthrough is accomplished,  $F = G = 0$ . A larger value of either  $F$  or  $G$  indicates lack of such sophistication. It is even possible to have a negative  $F$  or  $G$ , for example when the firm or the agent has to incur the cost of patenting a product. Even though we do not restrict our analysis with any assumption regarding the sign of  $F$  and  $G$ , we feel that usually  $(F + G)$  is nonnegative.

**D. The firm and the agent as decision makers.** We make the following assumptions regarding how the firm and agent make their decisions.

- (i) The agent knows the benefits he will receive from the environment if he selects the level of difficulty  $a$  and achieves a breakthrough. The firm knows the benefits it will receive as well as the benefits the agent will receive from the environment if there is a breakthrough. Both the firm and the agent know the function  $p(t, a)$ .
- (j) The firm is a risk neutral decision maker which knows the utility function of the agent, and its sole objective is to maximize net expected earnings from the project after compensating the agent.
- (k) The agent is risk neutral in earnings and his sole objective is to maximize expected utility from the project. His utility for income  $z$  and effort  $t$  is the additively separable function,  $z - V(t)$ , where  $V(t)$  is the disutility for effort  $t$ .
- (l)  $V'(t) > 0$ ,  $V''(t) \geq 0$ .

**E. The compensation plan for the agent.** The compensation the agent receives from the firm consists of (i) a constant base income,  $B$ , which does not depend on whether a breakthrough is achieved, and (ii) an additional amount, called a *reward* throughout the paper, which the agent receives only upon a breakthrough. Clearly, firms vary in their ability to relate reward to the quality of output. We make the following assumption about the firm.

- (m) The firm is either *naive* or *sophisticated*. The naive firm cannot relate reward to quality at all and gives the agent a fixed fee if a breakthrough occurs. In contrast, the sophisticated firm can base the reward on a perfect measure of quality of the output, and the reward the sophisticated firm gives the agent upon a breakthrough is a linear function of  $a$ .

Note that for the naive as well as the sophisticated firm, the agent's reward upon a breakthrough can be expressed as  $(s + qa)$ . The naive firm is restricted to using  $q = 0$ .

The nature of the output helps determine whether a firm is naive or sophisticated. If for example the agent is developing a commercial product, it may be possible to measure quality by monetary payoff and compensate the agent based on it. In contrast, if the agent is engaged in basic research, it is more difficult for the firm to achieve sophistication. The nature of the relationship between the firm and the agent is a factor as well, e.g., if the firm has to compensate the agent immediately after a breakthrough, it must be a quick judge of quality in order to be sophisticated.

Finally, most of the results in the paper are derived for the following specific functional forms for  $p(t, a)$  and  $V(t)$ :

$$(n) \quad p_1(t) = (1 - e^{-\alpha t}) \text{ and } p_2(a) = e^{-\beta a}, \alpha > 0, \beta > 0.$$

It can be easily verified that assumption (n) is consistent with assumptions (b) - (e).

$$(o) \quad V(t) = t.$$

Assumption (g) that the level of difficulty  $a$  becomes the quality upon breakthrough and assumption (m) that the sophisticated firm can use a perfect measure of  $a$  to reward the agent appear restrictive. However, since the firm and the agent are assumed to be risk neutral, the problem formulation is valid as long as quality upon breakthrough is an unbiased estimate of  $a$ , and the sophisticated firm can base the agent's reward on an unbiased estimate of quality and hence of  $a$ .

To summarize, we assume that the firm and the agent engage in a leader-follower game where the firm declares a compensation plan consisting of  $s$ ,  $q$ , and  $B$ , and the agent responds by selecting  $(t, a)$  to maximize his expected utility, called  $U(t, a)$  throughout this paper. The firm incorporates the agent's response in developing the optimal compensation plan, and its problem depends on whether it is naive, i.e., restricted to using  $q = 0$ , or sophisticated. The agent will agree to be employed by the firm only if he benefits from the relationship. Following the common practice in agency theory, we solve the firm's problem with a minimum expected utility constraint, i.e., the agent accepts employment if and only if his expected utility from it can equal or exceed a minimum, and the firm knows this minimum expected utility. We recognize that in practice, the firm is rather unlikely to know the exact value of the minimum expected utility the agent requires, and even the agent may be unsure of it. The agent may however still accept employment if the firm satisfies an alternative constraint to guarantee that the agent receives a fair share of the output. We later describe such a constraint, call the corresponding problem *the firm's problem without a minimum expected utility constraint*, and solve it as well. We therefore address the following four problems here:

- (i) the sophisticated firm without a minimum expected utility constraint,
- (ii) the sophisticated firm with a minimum expected utility constraint,
- (iii) the naive firm without a minimum expected utility constraint, and
- (iv) the naive firm with a minimum expected utility constraint.

The objectives of this study and the organization of the rest of the paper follow. In section 3, we analyze how an agent responds to a specific compensation plan offered in a given environment. The effect of changes in the reward mechanism and in parameters of the environment on the choice of  $(t, a)$  by the agent is discussed. In section 4 we develop the firm's problem of maximizing expected profit and formulate the problem for the four cases outlined above. A complete description of the objectives of the study of the firm's problem is also presented in section 4. Sections 5, 6, 7 and 8 present the problems of the sophisticated firm without a minimum expected utility constraint, the sophisticated firm with a minimum expected utility constraint, the naive firm without a minimum expected utility constraint, and the naive firm with a minimum expected utility constraint, respectively. In each case, we discuss the nature of the optimal compensation plan and how changes in parameters of the environment affect the optimal results. The relative profits of the naive and the sophisticated firm and the levels of quality they obtain are examined.

Only the results, sometimes augmented by a brief discussion of the rationale, are presented in the sections. The proofs of the results in sections 3, 5, 6, 7, and 8, as well as additional results to facilitate the proofs, are included in technical appendices A, B & C, D, E, and F respectively, and are available from the authors upon request.

Section 9 summarizes the findings of the paper and outlines directions for future research.

### 3. The Agent's Problem.

**3.1. General results.** The agent receives  $B$  from the firm regardless of whether a breakthrough is achieved. If he selects  $(t, a)$  and achieves a breakthrough, the net present value of his earnings from the firm and environment which result from the breakthrough is given by  $(s + qa + G + Qa)$ , i.e.,  $\{(G + s) + (Q + q)a\}$ . Since the agent is risk neutral, he chooses  $(t, a)$  to maximize his expected utility  $U(t, a)$  given by

$$(1) \quad U(t, a) = B + p(t, a)\{(G + s) + (Q + q)a\} - V(t).$$

Since  $U(t, a)$  does not depend on  $F$ , a change in  $F$  has no effect on the agent's choice of  $(t, a)$ . Also,  $B$  is a constant additive term in the expression for  $U(t, a)$  and changes in  $B$  have no influence on the agent's choice of  $(t, a)$ .

We first present three propositions regarding the agent's choice of  $(t, a)$  under assumptions (a) - (m) without making the more restrictive assumptions (n) and (o) about the functional forms of  $p(t, a)$  and  $V(t)$ , respectively.

**PROPOSITION 1.** *There always exists  $(t, a)$  where  $U(t, a)$  attains a strict global maximum. The global maximum occurs at a unique  $t$ . If this  $t$  exceeds zero, there is a unique  $a$  where the global maximum occurs. If this  $t$  equals zero, any  $a$  is globally optimal for the agent.*

In general, the agent's choice of  $(t, a)$  may be interior, i.e.,  $t > 0$  and  $a > 0$ , or boundary where one or both of  $t$  and  $a$  is zero. Proposition 2 presents the necessary and sufficient conditions for an interior solution to the agent's problem, and proposition 3 outlines how changes in the compensation plan or the parameters of the environment affect the solution.

**PROPOSITION 2.** *Consider  $(\tilde{t}, \tilde{a})$  such that  $\tilde{t} > 0$  and  $\tilde{a} > 0$ . Then,  $(\tilde{t}, \tilde{a})$  is the unique strict global optimum for the agent iff  $U_t = 0$  and  $U_a = 0$ .*

**PROPOSITION 3.** *Let  $(t, a)$  be the interior globally optimal solution to the agent's problem. Then,*

- (i)  $\frac{\partial t}{\partial s} = \frac{\partial t}{\partial G} > 0$ .     $\frac{\partial a}{\partial s} = \frac{\partial a}{\partial G} < 0$ .     $\frac{\partial p}{\partial s} = \frac{\partial p}{\partial G} > 0$ .
- (ii)  $\frac{\partial t}{\partial q} = \frac{\partial t}{\partial Q} > 0$ .     $\frac{\partial a}{\partial q} = \frac{\partial a}{\partial Q}$ , and it has the same sign as  $(G + s)$ .
- (iii)  $t$  and  $a$  do not depend on  $F$  or  $B$ .

If the fixed (quality independent) fee  $s$  increases, the agent finds it more important to attain a breakthrough. As a consequence,  $t$  increases but  $a$  decreases, i.e., the agent devotes more effort but reduces the level of difficulty attempted, thereby increasing the probability of breakthrough. An increase in  $Q$  induces a higher level of effort  $t$  from the agent. Also, if  $(G + s)$ , the total fixed reward the agent receives upon a breakthrough, is positive, the agent responds to an increase in reputation parameter by selecting a higher level of difficulty which translates into higher quality if there is a breakthrough. Since  $Q$  and  $q$  enter  $U(t, a)$  the same way, the effect of changes in  $q$  on the agent's choice of  $(t, a)$  is identical to the effect of changes in  $Q$  on those quantities. Similarly, the effect of a change in  $G$  on the agent's choice of  $(t, a)$  (and hence on  $p(t, a)$ ) is identical to that of a change in  $s$ .

**3.2. A special case.** In the remainder of section 3 and in sections 4, 5, 6, 7, and 8, we will make the additional assumptions (n) and (o). As a consequence, the agent's problem of finding  $(t, a)$  can be solved explicitly, which in turn facilitates the analysis of the firm's problem of finding the optimal compensation plan. The following proposition presents the globally optimal solution to the agent's problem under the full set of assumptions (a) - (o).

**PROPOSITION 4.** *The agent's choice of  $(t, a)$  and the corresponding value of  $p(t, a)$  are as follows:*

- (1) *If  $(Q + q) \leq 0$  or  $(G + s) \geq \frac{Q+q}{\beta}$ , then the agent's choice of  $a$  can WLOG be set equal to zero and the following two cases are possible regarding  $t$  and  $p(t, a)$ .*

**Case (i) : If  $(G + s) \leq \frac{1}{\alpha}$ , then  $t = 0$  and  $p(t, a) = 0$ .**

**Case (ii) : If  $(G + s) > \frac{1}{\alpha}$ , then**

$$(2) \quad t = \frac{1}{\alpha} \ln\{\alpha(G + s)\} > 0, \quad \text{and} \quad p(t, a) = 1 - \frac{1}{\alpha(G + s)} > 0.$$

**(2) If  $(Q + q) > 0$  and  $(G + s) < \frac{Q+q}{\beta}$ , then the agent's choice of  $a$  can WLOG be set equal to  $\frac{1}{\beta} - \frac{(G+s)}{(Q+q)} > 0$ , and the following two cases are possible regarding  $t$  and  $p(t, a)$ .**

**Case (i) : If  $e^{-1+\beta(G+s)/(Q+q)} \leq \frac{\beta}{\alpha(Q+q)}$ , then  $t = 0$  and  $p(t, a) = 0$ .**

**Case (ii) : If  $e^{-1+\beta(G+s)/(Q+q)} > \frac{\beta}{\alpha(Q+q)}$ , then**

$$(3) \quad t = \frac{1}{\alpha} \ln \left\{ \frac{\alpha(Q+q)}{\beta} e^{-1+\beta(G+s)/(Q+q)} \right\} > 0,$$

and

$$(4) \quad p(t, a) = e^{-1+\beta(G+s)/(Q+q)} - \frac{\beta}{\alpha(Q+q)} > 0.$$

If the agent selects  $t = 0$ , i.e., abandons the project, the choice of  $a$  does not have any material effect on the results of the firm or the agent. This corresponds to the "WLOG" in the statement of proposition 4. It is interesting to note that the agent selects  $t > 0$  if and only if at least one of  $(G + s)$  and  $(Q + q)$  is sufficiently large. The following four sections use the agent's response to a compensation plan given by proposition 4 to determine the optimal compensation plan of the firm.

#### 4. The Firm's Problem : An Overview

The firm pays the agent  $B$  regardless of whether a breakthrough occurs. If there is a breakthrough, the net present value of the firm's earnings from the environment is  $(F + a)$ , and it has to pay the agent  $(s + qa)$ . Therefore, the firm's expected profit is

$$(5) \quad \pi = p(t, a) \{(F + a) - (s + qa)\} - B = p(t, a) \{(F - s) + (1 - q)a\} - B$$

where  $t$ ,  $a$  and hence  $p(t, a)$  are given by proposition 4. The firm's problem is to select values of decision variables ( $s$ ,  $q$  &  $B$  for the sophisticated firm, and  $s$  &  $B$  for the naive firm) to maximize  $\pi$ .

Suppose the firm has selected  $s$  and  $q$  (only these affect the agent's choice of effort and difficulty), and the agent has selected  $(t, a)$  in response. If there is a breakthrough, the system consisting of the firm and the agent receives the fixed reward  $(F + G)$  and the quality

dependent reward  $(1 + Q)a$ , and the system expected earnings, denoted by  $\psi$  throughout this paper, is given by

$$(6) \quad \psi = p(t, a)\{(F + G) + (1 + Q)a\}.$$

We first formulate the firm's problem without a minimum expected utility constraint. The agent's base income  $B$  does not affect his choice of action and is likely to be determined by negotiations between the firm and the agent based on prevalent industry practice, etc. We arbitrarily set it equal to zero, i.e., neither the firm nor the agent earns anything from the project unless there is a breakthrough. With this specification, the problems of the sophisticated and the naive firm without a minimum expected utility constraint are as follows.

### S1. The sophisticated firm without a minimum expected utility constraint.

$$(7) \quad \max_{s, q} \pi \quad \text{where} \quad \pi = (1 - e^{-\alpha t})e^{-\beta a}(F - s + (1 - q)a),$$

subject to the constraint that the agent selects  $(t, a)$  to maximize his own expected utility, i.e.,

$$(8) \quad (t, a) \in \operatorname{argmax} \{(1 - e^{-\alpha t})e^{-\beta a}[(G + s) + (Q + q)a] - t\}.$$

### N1. The naive firm without a minimum expected utility constraint.

$$(9) \quad \max_s \pi \quad \text{where} \quad \pi = (1 - e^{-\alpha t})e^{-\beta a}(F - s + a),$$

subject to the constraint that the agent selects  $(t, a)$  to maximize his own expected utility, i.e.,

$$(10) \quad (t, a) \in \operatorname{argmax} \{(1 - e^{-\alpha t})e^{-\beta a}[(G + s) + Qa] - t\}.$$

Suppose now the firm must satisfy the constraint that the agent's expected utility must equal or exceed a minimum,  $m$ . Since the agent's choice of  $(t, a)$  does not depend on  $B$ , here the firm can always leave  $(s, q)$  the same and adjust  $B$  such that the agent's expected utility is exactly  $m$ . Since we have assumed that  $V(t) = t$  (assumption (o)), this implies that if the firm declares  $(s, q)$  which induces  $(t, a)$  from the agent, it also sets  $B$  such that the agent's expected earnings is  $(m + t)$ , and the remainder of the expected system earnings goes to the firm. Therefore, in this case,

$$(11) \quad \pi = \psi - (m + t) = p(t, a)\{(F + G) + (1 + Q)a\} - (m + t).$$

Therefore, if the firm must satisfy a minimum expected utility constraint of the agent, the problems for the sophisticated firm and the naive firm are as follows :

## S2. The sophisticated firm with a minimum expected utility constraint.

$$(12) \quad \max_{s,q} \pi \text{ where } \pi = (1 - e^{-\alpha t})e^{-\beta a} \{(F + G) + (1 + Q)a\} - m - t,$$

subject to the constraint that the agent selects  $(t, a)$  to maximize his own expected utility, i.e.,

$$(13) \quad (t, a) \in \operatorname{argmax} \{(1 - e^{-\alpha t})e^{-\beta a}[(G + s) + (Q + q)a] - t\}.$$

## N2. The naive firm with a minimum expected utility constraint.

$$(14) \quad \max_s \pi \text{ where } \pi = (1 - e^{-\alpha t})e^{-\beta a} \{(F + G) + (1 + Q)a\} - m - t,$$

subject to the constraint that the agent selects  $(t, a)$  to maximize his own expected utility, i.e.,

$$(15) \quad (t, a) \in \operatorname{argmax} \{(1 - e^{-\alpha t})e^{-\beta a}[(G + s) + Qa] - t\}.$$

It is important to emphasize that S1 and N1 are not relaxed versions of S2 and N2. Rather, they replace the constraint that the agent must be able to achieve  $U(t, a) \geq m$  by the constraint,  $B = 0$ .

We will generally restrict our attention to either a firm with a minimum expected utility constraint, or without a minimum expected utility constraint. Therefore, we will use the same set of notations to describe both cases. Throughout this paper, for a given problem, we mark optimal results with an asterisk. For example,  $(s^*, q^*)$  denotes the optimal compensation plan and  $\pi^*$  the expected profit of the firm generated by it, etc. When we discuss the relative performance of the naive and the sophisticated firm, the subscripts  $N$  and  $S$  are used.

In analyzing the four problems defined above, we focus on the issues listed below.

1. The nature of the optimal compensation plan in the different cases.
2. A comparison of the expected profits of the naive and the sophisticated firm. Since the naive firm must use  $q = 0$ ,  $\pi_N^* \leq \pi_S^*$ , and  $(\pi_S^* - \pi_N^*)$  represents the value of a firm's ability to relate compensation to quality. We identify cases where  $\pi_N^* = \pi_S^*$ .
3. A comparison of the levels of difficulty the naive and the sophisticated firm select.
4. Comparative statics to determine how the optimal results depend on the parameters of the environment,  $\alpha$ ,  $\beta$ ,  $F$ ,  $G$ , and  $Q$ . The specific effects studied vary from problem to problem. The following list describes the quantities studied for the sophisticated firm without a minimum expected utility constraint (problem S1).
  - a. The total fixed reward the agent receives upon a breakthrough,  $(G + s^*)$ . Since  $\frac{\partial s^*}{\partial G} = \frac{\partial(G+s^*)}{\partial G} - 1$ , and the partial derivative of  $(G + s^*)$  with respect to  $F$ ,  $\alpha$ ,  $\beta$  or  $Q$  is the same as the partial derivative of  $s^*$ , a study of  $(G + s^*)$  is sufficient to describe how  $s^*$  depends on the parameters of the environment. Similarly, the study of  $(G + s^*)$  also yields results for  $(F - s^*)$ , the total fixed reward of the firm upon a breakthrough.

b.  $(Q + q^*)$ , which determines the total quality dependent reward of the agent. Since  $\frac{\partial q^*}{\partial Q} = \frac{\partial(Q + q^*)}{\partial Q} - 1$ , and the partial derivative of  $(Q + q^*)$  with respect to  $\alpha, \beta, F$  or  $G$  is identical to the partial derivative of  $q^*$ , results for  $(Q + q^*)$  are sufficient to describe how  $q^*$  varies with  $\alpha, \beta, F, G$ , or  $Q$ . Similarly, results for  $(1 - q^*)$ , which determines the total quality dependent reward of the firm, follow from the analysis of  $(Q + q^*)$ .

- c. The level of difficulty  $a^*$  induced by the optimal compensation plan.
- d. The effort  $t^*$  induced by the optimal compensation plan.
- e. The probability of breakthrough  $p^* \equiv p(t^*, a^*)$ .
- f. The agent's expected reward  $z^* = p^* \{(G + s^*) + (Q + q^*)a\}$ .
- g. The agent's expected utility,  $U^* = z^* - t^*$ .
- h. The firm's expected profit  $\pi^*$ .
- i. The expected system earnings,  $\psi^* = p^* \{(F + G) + (1 + Q)a^*\}$ .
- j.  $\phi^*$ , the fraction of the system earnings which the agent receives if there is a breakthrough.  $\phi^*$  equals  $\frac{\{(G + s^*) + (Q + q^*)a^*\}}{\{(F + G) + (1 + Q)a^*\}}$ .

For the naive firm without a minimum expected utility constraint (problem N1), we study all the quantities listed above except  $(Q + q^*)$  which always equals  $Q$ .

Regarding a firm with a minimum expected utility constraint,  $U^*$  always equals  $m$ ,  $z^* = m + t^*$ , and the study of  $\phi^*$  is complicated by the presence of  $m$ . Therefore, for a sophisticated firm with a minimum expected utility constraint (problem S2), we only study  $(G + s^*)$ ,  $(Q + q^*)$ ,  $a^*$ ,  $t^*$ ,  $\pi^*$  and  $\psi^*$ . For a naive firm with a minimum expected utility constraint (problem N2), comparative statics is restricted to  $(G + s^*)$ ,  $a^*$ ,  $t^*$ ,  $\pi^*$  and  $\psi^*$ .

## 5. The Sophisticated Firm Without A Minimum Expected Utility Constraint

**5.1. Introduction.** In this case, the firm selects  $B = 0$  and solves the problem S1 described in section 4. The sophisticated firm can always construct a compensation plan which induces the agent to select  $t = 0$ , i.e., abandon the project, and hence generates  $\pi = 0$ .<sup>1</sup> Therefore,  $\pi_s^* \geq 0$ . The following two propositions present the necessary and sufficient conditions required to have  $\pi_s^* > 0$ , and outline the optimal compensation plan.

**PROPOSITION 5.** If  $(F + G) \geq \frac{1+Q}{\beta}$  then the sophisticated firm can generate  $\pi > 0$  iff  $(F + G) > \frac{1}{\alpha}$ . If  $(F + G) > \frac{1}{\alpha}$ , the optimal compensation plan  $(s^*, q^*)$  is as follows:

$$(16) \quad s^* = s^\dagger = \sqrt{(F + G)/\alpha} - G, \quad \text{and} \quad q^* \leq q^\dagger = \beta \sqrt{(F + G)/\alpha} - Q.$$

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<sup>1</sup>This is accomplished, for example, by a compensation plan which satisfies the following two conditions : (i)  $0 < \alpha(G + s) \leq 1$ , and (ii)  $\frac{Q+q}{\beta} \leq (G + s)$ .

This plan induces

$$(17) \quad a^* = 0, \quad \text{and} \quad t^* = \frac{1}{2\alpha} \ln\{\alpha(F + G)\} > 0.$$

Any other  $(s, q)$  generates a strictly lower  $\pi$ .

It is interesting to note that  $s^*$  is uniquely given by  $s^t$ , but any  $q \leq q^t$ , i.e., any  $q$  low enough to induce  $a = 0$  for  $s = s^t$ , is optimal. In the remainder of the paper, we always define  $s^t$  and  $q^t$  according to (16).

**PROPOSITION 6.** If  $(F + G) < \frac{1+Q}{\beta}$ , the sophisticated firm can generate  $\pi > 0$  iff the following condition is satisfied:

$$(18) \quad e^{-1+\beta(F+G)/(1+Q)} - \frac{\beta}{\alpha(1+Q)} > 0 \iff \left(\frac{1+Q}{\beta}\right)e^{-1+\beta(F+G)/(1+Q)} > \frac{1}{\alpha}.$$

If (18) is satisfied, the optimal compensation plan for the sophisticated firm is  $(s^*, q^*)$  where

$$(19) \quad q^* = \sqrt{\left(\frac{\beta e}{\alpha}\right)(1+Q)e^{-\beta(F+G)/(1+Q)}} - Q, \quad \text{and} \quad s^* = \left(\frac{F+G}{1+Q}\right)(Q + q^*) - G.$$

This plan generates a strictly greater  $\pi$  than any other  $(s, q)$ , and it induces

$$(20) \quad a^* = \frac{1}{\beta} - \frac{F+G}{1+Q} > 0, \quad \text{and} \quad t^* = \frac{1}{2\alpha} \ln \left[ \frac{\alpha(1+Q)}{\beta} e^{-1+\beta(F+G)/(1+Q)} \right] > 0.$$

We now focus on the case where the sophisticated firm can generate  $\pi > 0$ , and present results separately for : (i)  $(F + G) \geq \frac{1+Q}{\beta}$  and  $(F + G) > \frac{1}{\alpha}$ , i.e.,  $a^* = 0$ , and (ii)  $(F + G) < \frac{1+Q}{\beta}$  and (18) is satisfied, i.e.,  $a^* > 0$ .

**5.2. Results and comparative statics, case 1 :**  $(F + G) \geq \frac{1+Q}{\beta}$ , and  $(F + G) > \frac{1}{\alpha}$ . In this case,  $a^* = 0$  and  $s^*, q^*$  and  $t^*$  are given in the statement of proposition 5. The other quantities of interest are :

$$(21) \quad 0 < (G + s^*) = \sqrt{(F + G)/\alpha} < (F + G)$$

$$(22) \quad (Q + q^*) \leq \beta \sqrt{(F + G)/\alpha},$$

$$(23) \quad p^* = 1 - \frac{1}{\sqrt{\alpha(F + G)}} > 0,$$

$$(24) \quad z^* = \sqrt{(F+G)/\alpha} - \frac{1}{\alpha} > 0,$$

$$(25) \quad U^* = z^* - t^* > 0,$$

$$(26) \quad \pi^* = \left( \sqrt{F+G} - \frac{1}{\sqrt{\alpha}} \right)^2 = \alpha z^{*2} > 0,$$

$$(27) \quad \psi^* = p^*(F+G) = (F+G) - \sqrt{(F+G)/\alpha} > 0,$$

and

$$(28) \quad 0 < \phi^* = \frac{(G+s^*)}{(F+G)} = \frac{1}{\sqrt{\alpha(F+G)}} < 1.$$

The signs of  $(G+s^*)$ ,  $t^*$ ,  $p^*$ ,  $z^*$ ,  $U^*$ ,  $\pi^*$ ,  $\psi^*$  and  $\phi^*$ , and the result that  $(G+s^*) < (F+G)$  all follow from the fact that  $(F+G) > \frac{1}{\alpha}$ . We now restrict our attention to the case where  $(F+G) > \frac{1+Q}{\beta}$ , i.e., an infinitesimal change in parameter values does not alter the form of the optimal compensation plan, and present the results of comparative statics as propositions 7 - 9.

**PROPOSITION 7.** *If  $(F+G) > \frac{1+Q}{\beta}$  and  $(F+G) > \frac{1}{\alpha}$ , then changes in  $\beta$  and  $Q$  have no impact on the optimal results of the sophisticated firm.*

Intuitively, since  $a^* = 0$  and remains so for minute changes in parameter values, changes in  $\beta$  do not affect the probability of a breakthrough and hence have no impact on the results. Similarly, here the quality dependent benefit is zero, which eliminates the effect of  $Q$ .

**PROPOSITION 8.** *If  $(F+G) > \frac{1+Q}{\beta}$  and  $(F+G) > \frac{1}{\alpha}$ , then changes in  $\alpha$  affect the optimal solution as follows :*

(a)  $\frac{\partial(G+s^*)}{\partial\alpha}$  and  $\frac{\partial\phi^*}{\partial\alpha}$  are strictly negative.

(b)  $\frac{\partial p^*}{\partial\alpha}$ ,  $\frac{\partial\pi^*}{\partial\alpha}$  and  $\frac{\partial\psi^*}{\partial\alpha}$  are strictly positive.

(c)  $\frac{\partial t^*}{\partial\alpha}$ ,  $\frac{\partial z^*}{\partial\alpha}$  and  $\frac{\partial U^*}{\partial\alpha}$  may be positive or negative.  $\text{sgn}(\frac{\partial t^*}{\partial\alpha}) = \text{sgn}(e - \alpha(F+G))$ ,  $\text{sgn}(\frac{\partial z^*}{\partial\alpha}) = \text{sgn}(4 - \alpha(F+G))$ , and  $\text{sgn}(\frac{\partial U^*}{\partial\alpha}) = \text{sgn}(12.34 - \alpha(F+G))$ .

Intuitively, an increase in  $\alpha$  increases productivity by enhancing the probability of breakthrough for a given level of effort. When  $\alpha$  is low, an increase in  $\alpha$  leads the firm to try to induce a larger  $t$ , which can be accomplished at a lower incremental cost. However, when  $\alpha$  is large, the probability of breakthrough is already high, and an increase in  $\alpha$  provides the firm with an opportunity to reduce  $t$  and hence cost without losing productivity. As a consequence,  $z^*$  decreases when  $\alpha$  is large.  $U^*$  also decreases when  $\alpha$  is large, but this occurs at very high levels of  $\alpha$  since  $z^*$  and  $t^*$  have opposite effects on  $U^*$ . Also, as  $\alpha$  increases, the system expected benefit increases, and the firm absorbs a progressively larger share of it, thereby reducing the agent's share  $\phi^*$ .

**PROPOSITION 9.** If  $(F + G) > \frac{1+Q}{\beta}$  and  $(F + G) > \frac{1}{\alpha}$ , then  $F$  and  $G$  enter the expressions for  $(G + s^*)$ ,  $t^*$ ,  $p^*$ ,  $z^*$ ,  $U^*$ ,  $\pi^*$  and  $\psi^*$  through the sum,  $(F + G)$ , and the following hold.

(a)  $(G + s^*)$ ,  $t^*$ ,  $p^*$ ,  $z^*$ , and  $U^*$  are strictly increasing, strictly concave functions of  $(F + G)$ . Also,  $\frac{\partial(G + s^*)}{\partial(F + G)} < \frac{1}{2}$ ,  $\frac{\partial z^*}{\partial(F + G)} < \frac{1}{2}$ , and as  $(F + G) \rightarrow \infty$ ,  $p^* \rightarrow 1$ .

(b)  $\frac{\partial \pi^*}{\partial(F + G)} = p^* > 0$  and  $0 < p^* < \frac{\partial \psi^*}{\partial(F + G)} < 1$ . Both  $\pi^*$  and  $\psi^*$  are strictly convex in  $(F + G)$ .

(c)  $\phi^*$  is a strictly decreasing, strictly convex function of  $(F + G)$  and as  $(F + G) \rightarrow \infty$ ,  $\phi^* \rightarrow 0$ .

From proposition 9 it follows that as the system fixed reward increases, the agent receives less than half of the increase through the fixed reward  $(G + s^*)$  which increases at a diminishing rate. The firm's share increases with  $(F + G)$ . When  $(F + G)$  is large,  $p^* \approx 1$ , and the firm receives almost all of the increment in the system expected benefit.

**5.3. Results and comparative statics, case 2 :**  $(F + G) < \frac{1+Q}{\beta}$ , and (18) is satisfied. In this case,  $a^* > 0$  and  $s^*$ ,  $q^*$ , and  $t^*$  are given in the statement of proposition 6. The other quantities of interest are :

$$(29) \quad G + s^* = \frac{(F + G)}{(1+Q)} \sqrt{\frac{\beta(1+Q)}{\alpha} e^{1-\beta(F+G)/(1+Q)}} \quad \text{and} \quad \operatorname{sgn}(G + s^*) = \operatorname{sgn}(F + G),$$

$$(30) \quad 0 < Q + q^* = \sqrt{\frac{\beta(1+Q)}{\alpha} e^{1-\beta(F+G)/(1+Q)}} < (1+Q),$$

$$(31) \quad p^* = e^{-1+\beta(F+G)/(1+Q)} - \sqrt{\frac{\beta}{\alpha(1+Q)} e^{-1+\beta(F+G)/(1+Q)}} > 0,$$

$$(32) \quad z^* = \sqrt{\left(\frac{1+Q}{\alpha\beta}\right) e^{-1+\beta(F+G)/(1+Q)}} - \frac{1}{\alpha} > 0,$$

$$(33) \quad U^* = z^* - t^* > 0,$$

$$(34) \quad \pi^* = \left[ \sqrt{\left(\frac{1+Q}{\beta}\right) e^{-1+\beta(F+G)/(1+Q)}} - \frac{1}{\sqrt{\alpha}} \right]^2 = \alpha z^{*2} > 0,$$

$$(35) \quad \psi^* = \left(\frac{1+Q}{\beta}\right) p^* > 0,$$

$$(36) \quad 0 < \phi^* = \sqrt{\frac{\beta}{\alpha(1+Q)} e^{-1+\beta(F+G)/(1+Q)}} = \frac{G+s^*}{F+G} = \frac{Q+q^*}{1+Q} < 1$$

In this case the quality dependent reward of the system upon a breakthrough is  $(1+Q)a^*$  of which the firm receives  $(1-q^*)a^*$  and the agent receives  $(Q+q^*)a^*$ . It follows from (18) and (19) that  $q^* < 1$ , i.e.,  $(1-q^*) > 0$ . Since  $(Q+q^*) > 0$  as well, both the firm and the agent receive a share of the quality dependent system reward. It is interesting to note that when  $Q$  is large,  $q^* < 0$ , i.e., the sophisticated firm imposes a quality dependent ‘tax’ on the agent. From (36),  $(F-s^*)$  and  $(G+s^*)$ , the firm’s and the agent’s fixed reward upon breakthrough, both have the same sign as  $(F+G)$ . If  $(F+G) > 0$ , both parties share the benefit, while if  $(F+G) < 0$ , e.g., when a cost of patenting an innovation is involved, both parties share that cost. It also follows from (36) that under the optimal contract, the fraction of the system fixed reward the agent receives if there is a breakthrough is equal to the corresponding fraction of the system quality dependent reward.

The results of the comparative statics are summarized in propositions 10 - 13.

**PROPOSITION 10.** *If  $(F+G) < \frac{1+Q}{\beta}$  and (18) is satisfied, then the optimal results are affected by changes in  $\alpha$  as follows :*

1.  $\frac{\partial(G+s^*)}{\partial\alpha} < 0$ ,  $\frac{\partial(Q+q^*)}{\partial\alpha} < 0$ , and  $\frac{\partial\phi^*}{\partial\alpha} < 0$ .
2.  $a^*$  is not affected by changes in  $\alpha$ .
3.  $\frac{\partial p^*}{\partial\alpha}$ ,  $\frac{\partial\pi^*}{\partial\alpha}$ , and  $\frac{\partial\psi^*}{\partial\alpha}$  are all strictly positive.
4.  $\frac{\partial t^*}{\partial\alpha}$ ,  $\frac{\partial z^*}{\partial\alpha}$ , and  $\frac{\partial U^*}{\partial\alpha}$  may be positive or negative.

As in the case when  $a^* = 0$ , here an increase in  $\alpha$  leads to an increase in the system expected benefit, but the share of the agent declines.

**PROPOSITION 11.** *If  $(F+G) < \frac{1+Q}{\beta}$  and (18) is satisfied, then the optimal results are affected by changes in  $\beta$  as follows :*

1.  $\frac{\partial(G+s^*)}{\partial\beta} = \frac{1}{2}a^*(G+s^*) > 0$ ,  $\frac{\partial(Q+q^*)}{\partial\beta} = \frac{1}{2}a^*(Q+q^*) > 0$ .
2.  $\frac{\partial a^*}{\partial\beta} < 0$ ,  $\frac{\partial t^*}{\partial\beta} < 0$ , and  $\frac{\partial p^*}{\partial\beta}$  may be positive or negative.
3.  $\frac{\partial z^*}{\partial\beta} < 0$ ,  $\frac{\partial U^*}{\partial\beta} < 0$ , and  $\frac{\partial\phi^*}{\partial\beta} > 0$ .
4.  $\frac{\partial\pi^*}{\partial\beta} < 0$  and  $\frac{\partial\psi^*}{\partial\beta} < 0$ .

Intuitively, an increase in  $\beta$  reduces the probability of breakthrough for any given level of difficulty above the lowest, and the sophisticated firm responds by inducing a lower level of difficulty. The system expected benefit  $\psi^*$  declines and the agent’s share of it increases, both to the detriment of the firm’s expected profit. Interestingly,  $(G+s^*)$  and  $(Q+q^*)$  both increase with  $\beta$  and with the same elasticity.

**PROPOSITION 12.** If  $(F + G) < \frac{1+Q}{\beta}$  and (18) is satisfied,  $F$  and  $G$  enter the expressions for  $(G + s^*)$ ,  $(Q + q^*)$ ,  $a^*$ ,  $t^*$ ,  $p^*$ ,  $z^*$ ,  $U^*$ ,  $\pi^*$  and  $\psi^*$  through the sum,  $(F + G)$ , and the optimal results are affected by changes in  $(F + G)$  as follows:

1.  $0 < \frac{\partial(G+s^*)}{\partial(F+G)} < 1$ , and  $\frac{\partial(Q+q^*)}{\partial(F+G)} < 0$ .
2.  $\frac{\partial a^*}{\partial(F+G)} < 0$ ,  $\frac{\partial t^*}{\partial(F+G)} > 0$ , and  $\frac{\partial p^*}{\partial(F+G)} > 0$ .
3.  $\frac{\partial z^*}{\partial(F+G)} > 0$ ,  $\frac{\partial U^*}{\partial(F+G)} > 0$ , and  $\frac{\partial \phi^*}{\partial(F+G)} < 0$ .
4.  $\frac{\partial \pi^*}{\partial(F+G)} = p^* > 0$ , and  $\frac{\partial \psi^*}{\partial(F+G)} > p^* > 0$ .

Intuitively, as  $(F + G)$  increases, the system receives a greater fixed reward from the environment if a breakthrough is achieved. Therefore, the sophisticated firm modifies the compensation plan to increase the fixed reward and decrease the quality dependent reward of the agent in order to increase the probability of breakthrough. The expected earnings of both the firm and the agent increase, and the firm absorbs a progressively greater share of the system benefits.

**PROPOSITION 13.** If  $(F + G) < \frac{1+Q}{\beta}$  and (18) is satisfied, then the optimal results are affected by changes in  $Q$  as follows :

1.  $\frac{\partial(G+s^*)}{\partial Q} < 0$ , and  $0 < \frac{\partial(Q+q^*)}{\partial Q} < 1$ .
2.  $\frac{\partial a^*}{\partial Q} = (F+G)/(1+Q)^2$ , and  $\text{sgn}(\frac{\partial a^*}{\partial Q}) = \text{sgn}(F+G)$ .
3.  $\frac{\partial t^*}{\partial Q} > 0$  and  $\frac{\partial p^*}{\partial Q}$  may be positive or negative.
4.  $\frac{\partial z^*}{\partial Q} > 0$ ,  $\frac{\partial U^*}{\partial Q} > 0$ , but  $\frac{\partial \phi^*}{\partial Q} < 0$ .
5.  $\frac{\partial \pi^*}{\partial Q} > 0$  and  $\frac{\partial \psi^*}{\partial Q} > 0$ .

It is interesting to note that here  $\frac{\partial q^*}{\partial Q} < 0$ , i.e., as  $Q$  increases, the sophisticated firm reduces  $q$  to retain or extract a greater portion of the quality dependent reward. The system expected benefit increases but the agent now receives a smaller share of it.

**5.4. Summary.** To summarize, we notice in both case 1 and case 2 that  $F$  and  $G$  enter the expressions for all the quantities of interest through their sum  $(F + G)$ , i.e., the system does not retain a memory of whether the fixed reward originates with the firm or the agent. We find from propositions 9 and 12 that when  $(F + G)$  increases, the increment in fixed reward is shared by the firm and the agent. If only  $F$  increases, the firm transfers a part of the increase to the agent, and if only  $G$  increases, the firm extracts a part of the increase from the agent.

When  $(F + G) < \frac{1+Q}{\beta}$ , proposition 6 implies that  $q^* < 0$  when  $Q$  is large and when  $Q$  is small,  $q^*$  is usually positive. Consider for example a researcher who is working for a university or the R & D division of a commercial firm and is trying to develop a product. In this case, the organization can be considered to be sophisticated since it can usually keep

track of the earnings from the product. The commercial firm is likely to receive the major part of the earnings from a breakthrough, i.e.,  $Q$  is low. Here our model suggests that the optimal plan will offer a share of future profits to the researcher. In contrast, a researcher working for a university is likely to be the immediate beneficiary of a breakthrough, i.e.,  $Q$  is high. Here our model suggests that the researcher will have to agree to pay the university a share of future earnings from a product developed while he is employed there. These results are consistent with practice.

An increase in  $Q$  enhances system benefits if  $a^* > 0$ . Since  $Qa^*$  comes to the agent originally, the firm reduces  $q$  to obtain a share of the increment in quality dependent reward. However, both  $(1 - q^*)$  and  $(Q + q^*)$  increase. Finally, an inspection of the expressions for  $t^*$ ,  $p^*$ ,  $z^*$ ,  $U^*$ ,  $\psi^*$  and  $\phi^*$  when  $a^* > 0$  shows that  $Q$  and  $\beta$  enter them through the term  $\frac{1+Q}{\beta}$ , which explains why changes in  $\beta$  and  $Q$  have exactly the opposite effects on these terms.

It follows from propositions 5 and 6 that even the sophisticated firm can generate  $\pi > 0$  only if at least one of  $(F + G)$  and  $Q$  is reasonably large. It is interesting to note that if the agent selects  $a = 0$ , the system receives  $(F + G)$  if there is a breakthrough. Also, it follows from proposition 6 that if  $a^* > 0$ , the system receives  $\frac{1+Q}{\beta}$  if there is a breakthrough. The comparison of  $(F + G)$  and  $\frac{1+Q}{\beta}$  determines the level of difficulty the sophisticated firm should try to induce. A larger  $(F + G)$  leads the sophisticated firm to be more certain of a breakthrough and therefore reduces quality. If  $(F + G)$  is negative, clearly the firm will like to induce a high quality to offset the costs. However, if  $(F + G)$  is too small, the firm is not able to generate  $\pi > 0$  and induces the agent to select  $t = 0$  which is identical to abandoning the project.

A comparison of proposition 4 with propositions 5 and 6 reveals the similarity between the response of an agent to a compensation plan  $(s, q)$  and an external environment and that of the system consisting of a sophisticated firm without a minimum expected utility requirement and its agent to the same environment. In particular, noting that  $(G + s)$  and  $(Q + q)$  for the agent parallel  $(F + G)$  and  $(1 + Q)$  for the system, respectively, it follows that the expression for  $a$  selected by the agent is identical to the expression for  $a^*$  selected by the system, and the condition under which the agent selects  $t > 0$  exactly parallels the condition under which the system obtains  $t^* > 0$ . However, if we replace  $(G + s^*)$  by  $(F + G)$  and  $(Q + q^*)$  by  $(1 + Q)$  in equation 2 or 3, we will obtain  $2t^*$ . This indicates that the system consisting of a sophisticated firm without a minimum expected utility constraint does not achieve perfect coordination such that it will respond to the external environment as a single entity.

## 6. The Sophisticated Firm With A Minimum Expected Utility Constraint

**6.1. Introduction and the optimal compensation plan.** As discussed earlier, the sophisticated firm with a minimum expected utility constraint solves problem S2 defined in section 4. It can easily be shown that if the firm sets  $s = F$  and  $q = 1$ , the objectives of the firm and the agent become identical and the constraint (13) corresponding to the agent's choice of  $(t, a)$  is effectively eliminated. The sophisticated firm in this case obtains the *first best* result. Here, the sophisticated firm finds it optimal to allow the agent to take the entire benefit of the breakthrough in exchange for a fixed payment which the agent has to make regardless of whether a breakthrough is achieved. Even though the firm *sells* the output to the agent, it still has to be able to evaluate the quality of the breakthrough if it occurs.

If the rewards from a breakthrough are not sufficient to motivate the agent, he selects  $t = 0$ . In that case,  $\pi = -m$ , and this is the lowest expected profit the sophisticated firm can get from the optimal compensation plan.

The next proposition formally presents the optimal compensation plan and the level of difficulty and effort it generates.

**PROPOSITION 14.** *The sophisticated firm with a minimum expected utility constraint will obtain optimal results with the compensation plan  $s = F$  and  $q = 1$ .*

*If  $(F + G) \geq \frac{1+Q}{\beta}$  then  $\pi_s^* > -m$  iff  $(F + G) > \frac{1}{\alpha}$ . If  $(F + G) > \frac{1}{\alpha}$  then  $(t^*, a^*)$  are as follows:*

$$(37) \quad a^* = 0, \quad \text{and} \quad t^* = \frac{1}{\alpha} \ln\{\alpha(F + G)\} > 0.$$

*If  $(F + G) < \frac{1+Q}{\beta}$  then  $\pi_s^* > -m$  iff (18) is satisfied. If (18) is satisfied then  $(t^*, a^*)$  are as follows:*

$$(38) \quad a^* = \frac{1}{\beta} - \frac{F + G}{1 + Q} > 0, \quad \text{and} \quad t^* = \frac{1}{\alpha} \ln \left\{ \frac{\alpha(1 + Q)}{\beta} e^{-1 + \beta(F + G)/(1 + Q)} \right\} > 0.$$

It may sometimes be possible for the firm to obtain optimal results using a compensation plan other than  $(F, 1)$ . For example, if  $(F + G) \geq \frac{1+Q}{\beta}$ , the firm will obtain optimal results if it uses  $s = F$  and any  $q$  low enough to induce  $a = 0$ . However, if  $\pi^* > -m$ , it can be shown that any choice of  $t$  or  $a$  other than those listed in proposition 14 will generate a strictly lower  $\pi$ . Therefore,  $a^*$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  are unique even if the optimal compensation plan is not.

A comparison of proposition 14 with propositions 5 and 6 shows that the sophisticated firm with a minimum expected utility constraint can generate  $\pi > -m$  if and only if the

sophisticated firm without the constraint can generate  $\pi > 0$ . Also,  $a^*$  is the same in both cases. Unless  $t^* = 0$ , under the optimal compensation plan a breakthrough generates the same system benefit in both cases :  $(F + G)$  if  $a^* = 0$  and  $\frac{1+Q}{\beta}$  if  $a^* > 0$ . However,  $t^*$  for the sophisticated firm with a minimum expected utility constraint is twice the  $t^*$  for the sophisticated firm without the constraint, i.e., unless the project is abandoned, the probability of breakthrough and hence the expected system benefit is higher for the sophisticated firm with a minimum expected utility constraint. This increase in productivity results directly from the fact that the firm and the agent here act as a single entity.

**6.2. Results, comparative statics and summary.** We only consider the case where  $\pi^* > -m$ , i.e., the agent is not induced to abandon the project. Since it is optimal to use  $(G + s^*) = (F + G)$  and  $(Q + q^*) = (1 + Q)$ , we restrict our attention to  $a^*$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$ , and  $\psi^*$  which are now presented separately for the cases where (i)  $a^* = 0$  and (ii)  $a^* > 0$ .

**Case 1.**  $a^* = 0$ .  $t^*$  is presented in the statement of proposition 14. The expressions for  $p^*$ ,  $\pi^*$  and  $\psi^*$  follow.

$$(39) \quad p^* = 1 - \frac{1}{\alpha(F + G)}.$$

$$(40) \quad \pi^* = (F + G) - \frac{1}{\alpha} - \frac{1}{\alpha} \ln\{\alpha(F + G)\} - m > -m.$$

$$(41) \quad \psi^* = (F + G)p^*.$$

**Case 2.**  $a^* > 0$ . Once again,  $t^*$  and  $a^*$  are presented in the statement of proposition 14, and the expressions for  $p^*$ ,  $\pi^*$  and  $\psi^*$  follow.

$$(42) \quad p^* = e^{-1+\beta(F+G)/(1+Q)} - \frac{\beta}{\alpha(1+Q)} > 0.$$

$$(43) \quad \pi^* = \left(\frac{1+Q}{\beta}\right)e^{-1+\beta(F+G)/(1+Q)} - \frac{1}{\alpha} \ln\left\{\frac{\alpha(1+Q)}{\beta}\right\} - \frac{\beta(F+G)}{\alpha(1+Q)} - m > -m.$$

$$(44) \quad \psi^* = \left(\frac{1+Q}{\beta}\right)p^* > 0.$$

For both cases (i.e.,  $a^* = 0$  and  $a^* > 0$ ), the results of comparative statics (regarding the dependence of  $t^*$ ,  $a^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  on  $\alpha$ ,  $\beta$ ,  $F$ ,  $G$  and  $Q$ ) here are qualitatively identical to those for the sophisticated firm without a minimum expected utility constraint, and are presented as propositions 15 and 16 for the sake of completeness.

**PROPOSITION 15.** If  $(F + G) > \frac{1+Q}{\beta}$  and  $(F + G) > \frac{1}{\alpha}$ , then the optimal results of the sophisticated firm are affected by changes in parameters of the environment as follows:

1.  $a^*$  is always zero.
2. Effect of  $\alpha$ .  $\frac{\partial p^*}{\partial \alpha}$ ,  $\frac{\partial \pi^*}{\partial \alpha}$  and  $\frac{\partial \psi^*}{\partial \alpha}$  are strictly positive.  $\frac{\partial t^*}{\partial \alpha}$  has the same sign as  $\{e - \alpha(F + G)\}$ .
3. Effect of  $F$  and  $G$ .  $F$  and  $G$  enter the expressions for  $a^*$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  through their sum,  $(F + G)$ .  $\frac{\partial a^*}{\partial(F+G)}$ ,  $\frac{\partial p^*}{\partial(F+G)}$ ,  $\frac{\partial \pi^*}{\partial(F+G)}$ , and  $\frac{\partial \psi^*}{\partial(F+G)}$  are all strictly positive, and  $\frac{\partial \pi^*}{\partial(F+G)} = p^*$ .
4. Changes in  $\beta$  and  $Q$  have no effect on the optimal results.

[ In proposition 15, we only consider  $(F + G) > \frac{1+Q}{\beta}$  to ensure the stability of the form of the solution.]

**PROPOSITION 16.** If  $(F + G) < \frac{1+Q}{\beta}$  and (18) is satisfied, then the optimal results of the sophisticated firm depend on parameters of the environment as follows :

1. Effect of  $\alpha$ .  $a^*$  does not depend on  $\alpha$ .  $\frac{\partial p^*}{\partial \alpha}$ ,  $\frac{\partial \pi^*}{\partial \alpha}$  and  $\frac{\partial \psi^*}{\partial \alpha}$  are all strictly positive.  $\frac{\partial t^*}{\partial \alpha}$  has the same sign as  $\{e^2 - \frac{\alpha(1+Q)}{\beta} e^{\beta(F+G)/(1+Q)}\}$ .
2. The effect of  $\beta$ .  $\frac{\partial a^*}{\partial \beta}$ ,  $\frac{\partial t^*}{\partial \beta}$ ,  $\frac{\partial \pi^*}{\partial \beta}$  and  $\frac{\partial \psi^*}{\partial \beta}$  are all strictly negative.  $\frac{\partial p^*}{\partial \beta}$  can be positive or negative.
3. Effect of  $F$  &  $G$ .  $F$  and  $G$  enter the expressions for  $a^*$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  through the sum,  $(F + G)$ .  $\frac{\partial a^*}{\partial(F+G)} < 0$ , and  $\frac{\partial t^*}{\partial(F+G)}$ ,  $\frac{\partial p^*}{\partial(F+G)}$ ,  $\frac{\partial \pi^*}{\partial(F+G)}$  and  $\frac{\partial \psi^*}{\partial(F+G)}$  are all strictly positive. Also,  $\frac{\partial \pi^*}{\partial(F+G)} = p^*$ .
4. Effect of  $Q$ .  $\frac{\partial t^*}{\partial Q}$ ,  $\frac{\partial \pi^*}{\partial Q}$  and  $\frac{\partial \psi^*}{\partial Q}$  are all strictly positive.  $\frac{\partial a^*}{\partial Q}$  has the same sign as  $(F + G)$ .  $\frac{\partial p^*}{\partial Q}$  can be positive or negative.

It is interesting to note that for a sophisticated firm with or without a minimum expected utility constraint,  $\frac{\partial \pi^*}{\partial(F+G)}$  always equals  $p^*$ . The sophisticated firm with the minimum expected utility constraint obtains a larger  $p^*$  and hence receives a greater share of any increment in the system fixed reward  $(F + G)$ .

The results of comparative statics for the sophisticated firm with and without the minimum expected utility constraint (which are qualitatively identical in the region of overlap) are presented in Table 1.

Table 1 about here

## 7. The Naive Firm Without A Minimum Expected Utility Constraint

**Introduction.** From section 4, the naive firm without a minimum expected utility constraint but restricted to using  $B = 0$  solves the problem N1. Our presentation of this case

will focus on the following issues : (a) the formulation of the optimal compensation plan, (b) the discussion of results and comparative statics in the different cases, and (c) the comparison of the results with those for the sophisticated firm without a minimum expected utility constraint, simply called the *sophisticated firm* throughout section 7. The principal objective of this comparison is to determine the relative profitability of the naive and the sophisticated firm, and a secondary objective is to compare the levels of difficulty a naive and a sophisticated firm choose to induce.

**7.2. The optimal compensation plan.** The naive firm has only one decision variable,  $s$ , and it can be shown that  $\pi$  is a continuous function of  $s$ . Also, it can be shown that  $s$  can always be set low enough to induce  $t = 0$  and hence generate  $\pi = 0$ , i.e.,  $\pi_N^* \geq 0$ . From proposition 4, the agent selects  $a = 0$  if  $Q = 0$ , and if  $Q > 0$ , then  $a > 0$  if  $s < (\frac{Q}{\beta} - G)$  and  $a = 0$  if  $s \geq (\frac{Q}{\beta} - G)$ . If  $\frac{\beta}{\alpha Q} \geq 1$ , it can be shown that the naive firm cannot simultaneously induce  $t > 0$  and  $a > 0$  and hence can obtain  $\pi > 0$  only if  $a = 0$ . The next proposition outlines the optimal compensation plan in that case.

**PROPOSITION 17.** *If  $\frac{\beta}{\alpha Q} \geq 1$  then  $\pi_N^* > 0$  iff  $(F + G) > \frac{1}{\alpha}$ .*

*If  $(F + G) > \frac{1}{\alpha}$ , then the optimal compensation plan is  $(s^*, 0)$  which induces  $a^* = 0$  and generates  $\pi^* = [\sqrt{F + G} - \frac{1}{\sqrt{\alpha}}]^2 > 0$ .*

*Any other compensation plan of the form  $(s, 0)$  yields a strictly inferior  $\pi$ .*

*If in addition  $(F + G) \geq \frac{1+Q}{\beta}$  then  $\pi_N^* = \pi_s^*$ . Otherwise  $\pi_N^* < \pi_s^*$ .*

Let us now consider the case where  $\frac{\beta}{\alpha Q} < 1$ , which can occur only if  $Q > 0$ . For  $Q > 0$ , throughout the remainder of this paper, we define  $s_1$  as follows:

$$(45) \quad e^{-1+\beta(G+s_1)/Q} = \frac{\beta}{\alpha Q} \iff s_1 = \left(\frac{Q}{\beta} - G\right) + \frac{Q}{\beta} \ln\left(\frac{\beta}{\alpha Q}\right).$$

Since  $\frac{\beta}{\alpha Q} < 1$ ,  $s_1 < (\frac{Q}{\beta} - G)$ , and it can be shown that the agent selects  $t = 0$  if  $s \leq s_1$ , and  $t > 0$  if  $s > s_1$ . Since  $\pi = 0$  if  $t = 0$ , the naive firm can obtain  $\pi > 0$  only if  $s > s_1$ . The next proposition states the condition the naive firm must satisfy in order to generate  $\pi > 0$  in this case.

**PROPOSITION 18.** *If  $\frac{\beta}{\alpha Q} < 1$ , the naive firm can construct a compensation plan which generates  $\pi > 0$  only if the following condition is satisfied:*

$$(46) \quad e^{\beta(F+G)/(1+Q)} > \frac{\beta}{\alpha Q} e^{Q/(1+Q)} \iff (F + G) > \frac{Q}{\beta} + \left(\frac{1+Q}{\beta}\right) \ln\left(\frac{\beta}{\alpha Q}\right).$$

*If (46) is satisfied, then the naive firm can construct a compensation plan which induces  $a > 0$  and generates  $\pi > 0$ .*

We now restrict our attention to the case where  $\frac{\beta}{\alpha Q} < 1$  and (46) is satisfied. Since  $\pi_N^* > 0$  here, it is sufficient to consider  $s > s_1$  in order to obtain the optimal compensation plan. It

can be shown that  $\frac{\partial \pi}{\partial s}$  is a continuous function of  $s$  if  $s \in (s_1, \frac{Q}{\beta} - G)$  and if  $s > (\frac{Q}{\beta} - G)$ , but it may not exist at  $s = \frac{Q}{\beta} - G$ . Consequently, it is convenient to analyze the two regions : (i)  $s_1 < s < \frac{Q}{\beta} - G$ , where  $a > 0$ , and (ii)  $s \geq (\frac{Q}{\beta} - G)$ , where  $a = 0$ , separately. The following results for these two regions will help motivate the formation of the special cases presented next.

1.  $\pi$  is either a strictly increasing or a unimodal function of  $s \in (s_1, \frac{Q}{\beta} - G)$ , and

$$\lim_{s \uparrow \frac{Q}{\beta} - G} \frac{\partial \pi}{\partial s} = \frac{\beta}{Q} \left[ (F + G) - \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{(1+Q)}{\alpha Q} \right\} \right].$$

If  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{(1+Q)}{\alpha Q} \right\}$  then  $\pi$  is strictly increasing in  $s$  in this region and the compensation plan  $(\frac{Q}{\beta} - G, 0)$  is strictly superior to any plan  $(s, 0)$  with  $s < (\frac{Q}{\beta} - G)$ .

If  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{(1+Q)}{\alpha Q} \right\}$  then  $\pi$  is a unimodal function of  $s$  for  $s \in (s_1, \frac{Q}{\beta} - G)$  and has a unique maximum at  $\bar{s} \in (s_1, \frac{Q}{\beta} - G)$  where  $\frac{\partial \pi}{\partial s} = 0$  and which is uniquely given by

$$(47) \quad \frac{\beta(1+Q)}{\alpha Q^2} = e^{-1+\beta(G+\bar{s})/Q} \left[ 1 - \frac{\beta(F+G)}{Q} + \frac{\beta(G+\bar{s})(1+Q)}{Q^2} \right].$$

In the rest of section 7, we always define  $\bar{s}$  by equation 47.

2.  $\pi$  is a strictly concave function of  $s$  for  $s > \frac{Q}{\beta} - G$ , and  $\lim_{s \downarrow \frac{Q}{\beta} - G} = \frac{\beta^2(F+G)}{\alpha Q^2} - 1$ .

If  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ ,  $\pi$  is uniquely maximized for  $s \geq \frac{Q}{\beta} - G$  at  $s^\dagger$ . Here  $s^\dagger > \frac{Q}{\beta} - G$ , and the plan  $(s^\dagger, 0)$  generates  $\pi > 0$ .

If  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ ,  $\pi$  is uniquely maximized for  $s \geq (\frac{Q}{\beta} - G)$  at  $s = \frac{Q}{\beta} - G$ .

Propositions 19 - 22 present the nature of the optimal compensation plan in the following four cases.

- (A)  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ ,
- (B)  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ ,
- (C)  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ ,
- (D)  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  and  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ .

Figure 1 illustrates how  $\pi$  depends on  $s$  in the four cases defined above.

Figure 1 about here

It is interesting to note that  $\pi$  is a unimodal function of  $s$  in cases (A), (B) and (D), and is bimodal in case (C). Since  $\frac{\beta}{\alpha Q} < 1$  here, it can be easily shown that (46) is satisfied in cases (A), (B) and (C). We therefore do not include (46) as a separate condition in the statements of the propositions for these cases.

**PROPOSITION 19.** If  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ , then it is optimal for the naive firm to use the compensation plan  $(s^\dagger, 0)$ ,  $s^\dagger > (\frac{Q}{\beta} - G)$ , which induces  $a^* = 0$  and generates  $\pi^* = (\sqrt{F+G} - \frac{1}{\sqrt{\alpha}})^2 > 0$ .

**Note 1.** If  $(F + G) \geq \frac{1+Q}{\beta}$ , i.e.,  $a_S^* = 0$ , the conditions  $\frac{\beta}{\alpha Q} < 1$  and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$  jointly imply that  $(F + G) > \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , i.e., the conditions of proposition 19 are satisfied. A comparison with proposition 5 shows that in that case, the naive firm and the sophisticated firm obtain the same results.

**Note 2.** The conditions of proposition 19 may be satisfied even when  $(F + G) < \frac{1+Q}{\beta}$ , as the next example shows, and in that case,  $\pi_N^* < \pi_S^*$ .

**Example.**  $\alpha = 1$ ,  $\beta = .4$ ,  $Q = .5$ . In this case, if  $3.5 \leq (F + G) < 3.75$ , then  $(F + G) < \frac{1+Q}{\beta}$  but the conditions of proposition 19 are satisfied.

**PROPOSITION 20.** If  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ , then it is optimal for the naive firm to use  $s^* = (\frac{Q}{\beta} - G)$  which induces  $a^* = 0$ .

**Note 1.** It can be easily shown that the conditions of proposition 20 can be satisfied only if  $(F + G) \geq \frac{1+Q}{\beta}$ , i.e.,  $a_S^* = a_N^* = 0$ .

**Note 2.** If  $(F + G) < \frac{\alpha Q^2}{\beta^2}$  here, then  $\pi_N^* < \pi_S^*$ . In this case, the optimal compensation plan for the sophisticated firm is  $(s^\dagger, q)$  where  $s^\dagger < (\frac{Q}{\beta} - G)$  and  $q \leq q^\dagger < 0$ , i.e., the sophisticated firm must impose a “tax” on quality to dissuade the agent from attempting a difficult project. The naive firm can only use a higher fixed reward to induce  $a = 0$  and that reduces  $\pi$ .

**Note 3.** It can be easily shown that if  $(F + G) \geq \frac{1+Q}{\beta}$ ,  $(F + G) > \frac{1}{\alpha}$  and  $(F + G) = \frac{\alpha Q^2}{\beta^2}$ , then the conditions of proposition 20 are satisfied. Then and only then the compensation plan  $(\frac{Q}{\beta} - G, 0)$  generates  $\pi_N^* = \pi_S^* > 0$ .

**PROPOSITION 21.** If  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ , then  $\pi$  is a bimodal function of  $s$  with the modes at  $\bar{s} < (\frac{Q}{\beta} - G)$  and at  $s^\dagger > (\frac{Q}{\beta} - G)$ , and  $\exists \eta \in \left( \frac{\alpha Q^2}{\beta^2}, \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\} \right)$  such that if  $(F + G) = \eta$  then  $(\bar{s}, 0)$  and  $(s^\dagger, 0)$  generate the same value of  $\pi$ . If  $(F + G) < \eta$ ,  $(\bar{s}, 0)$  is strictly superior, and if  $(F + G) > \eta$ , then  $(s^\dagger, 0)$  is strictly superior.

It can be easily shown that the conditions of proposition 21 can be satisfied only if  $(F + G) < \frac{1+Q}{\beta}$ , i.e.,  $a_S^* > 0$ . If  $(F + G) > \eta$  here, then  $a_N^* = 0$  and  $\pi_N^* < \pi_S^*$ .

**Numerical Example.** Consider  $\alpha = 1$ ,  $\beta = .4$ , and  $Q = .5$ , which satisfies the condition  $\frac{\beta}{\alpha Q} < 1$ . Here,  $\frac{\alpha Q^2}{\beta^2} = 1.5625$  and  $\left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{(1+Q)}{\alpha Q} \right\} = 2$ .

Table 2 lists the naive firm’s expected profit at  $s = s^\dagger$  and  $s = \bar{s}$ ,  $\pi^*$  and  $a^*$ ,  $t^*$  and  $p^*$  for  $(F + G) = 1.6, 1.65, \dots, 1.95$ . From this table, it appears that as  $(F + G)$  increases,  $\pi$  for

both the modes increase, but  $\pi$  for  $(s^t, 0)$  increases faster.<sup>2</sup> A detailed numerical analysis shows that here,  $\eta \approx 1.75373$ . It is interesting to note that  $t^*$  and  $p^*$  increase sharply as  $(F + G)$  exceeds  $\eta$ .

Table 2 about here

**PROPOSITION 22.** If  $\frac{\beta}{\alpha Q} < 1$ , (46) is satisfied,  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ , then the optimal compensation plan for the naive firm is  $(\bar{s}, 0)$  where  $\bar{s} < (\frac{Q}{\beta} - G)$ , and  $a^* > 0$ .

**Note 1.** It can be shown that the conditions  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$  and  $(F + G) < \frac{1+Q}{\beta}$  jointly imply that  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ . If (46) is also satisfied, all the requirements of proposition 22 are fulfilled.

**Note 2.** The conditions of proposition 22 can be fulfilled even when  $(F + G) \geq \frac{1+Q}{\beta}$ , i.e.,  $a_s^* = 0$ , as the next example shows, and in that case  $\pi_N^* < \pi_S^*$ . This can occur only if  $(F + G) < \frac{\alpha Q^2}{\beta^2}$ , which implies that the sophisticated firm would use  $q < 0$ . The inability of the naive firm to do that hurts its profits.

**Example.** Let  $\alpha = 1$ ,  $\beta = .1$ , and  $Q = 1$ . If  $20 \leq (F + G) < 28$  then  $(F + G) \geq \frac{1+Q}{\beta}$  but the conditions of proposition 22 are satisfied.

**7.3. Results and comparative statics.** In this section, we will only consider cases where  $\pi_N^* > 0$ , i.e., one of the following two conditions is satisfied:

(i)  $\frac{\beta}{\alpha Q} \geq 1$  and  $(F + G) > \frac{1}{\alpha}$ , and (ii)  $\frac{\beta}{\alpha Q} < 1$  and (46) is satisfied.

From propositions 17, 19, 20, 21 and 22, we identify the following three distinct forms of the optimal compensation plan for the naive firm without a minimum expected utility constraint:

(I)  $s^* > \frac{Q}{\beta} - G$ , (II)  $s^* = (\frac{Q}{\beta} - G)$ , and (III)  $s^* < (\frac{Q}{\beta} - G)$ . We discuss these cases separately. The form of the solution may sometimes change from an infinitesimal variation in parameter values. The comparative statics presented are restricted to the cases where the form of the solution is stable.

**Case I.** In this case,  $s^* = s^t > \frac{Q}{\beta} - G$ , and this happens if

(i)  $\frac{\beta}{\alpha Q} \geq 1$  and  $(F + G) > \frac{1}{\alpha}$ , or

(ii)  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ .

This may also happen if the following condition is satisfied :

$\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) > \frac{\alpha Q^2}{\beta^2}$ .

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<sup>2</sup>This result is analytically derived in Appendix D and the proof of proposition 21 is based on it.

The expressions for  $s^*$ ,  $a^*$ ,  $t^*$ ,  $p^*$ ,  $z^*$ ,  $U^*$ ,  $\pi^*$ ,  $\psi^*$ , and  $\phi^*$  here are identical to those for the sophisticated firm when  $(F + G) \geq \frac{1+Q}{\beta}$  and presented in section 5.2. The results of comparative statics are also identical to those presented in section 5.2 and hence are not repeated here.

**Case II.** Here  $s^* = (\frac{Q}{\beta} - G)$  and this occurs if the following condition is satisfied :  
 $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) \geq \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$ , and  $(F + G) \leq \frac{\alpha Q^2}{\beta^2}$ ,

In this case,  $a^* = 0$  and the other quantities of interest are,

$$(48) \quad (G + s^*) = \frac{Q}{\beta} > 0,$$

$$(49) \quad t^* = \frac{1}{\alpha} \ln \left\{ \frac{\alpha Q}{\beta} \right\} > 0,$$

$$(50) \quad p^* = 1 - \frac{\beta}{\alpha Q} > 0,$$

$$(51) \quad z^* = \frac{Q}{\beta} - \frac{1}{\alpha} > 0,$$

$$(52) \quad U^* = \frac{Q}{\beta} - \frac{1}{\alpha} - \frac{1}{\alpha} \ln \left\{ \frac{\alpha Q}{\beta} \right\} > 0,$$

$$(53) \quad \pi^* = \left( 1 - \frac{\beta}{\alpha Q} \right) \left[ (F + G) - \frac{Q}{\beta} \right] > 0,$$

$$(54) \quad \psi^* = \left( 1 - \frac{\beta}{\alpha Q} \right) (F + G),$$

$$(55) \quad \phi^* = \frac{Q}{\beta(F + G)}.$$

If  $(F + G)$  equals either  $\left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  or  $\frac{\alpha Q^2}{\beta^2}$ , a slight change in any parameter of the environment can change the form of the solution. Therefore, we present comparative statics only for the case where  $(F + G) > \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}$  and  $(F + G) < \frac{\alpha Q^2}{\beta^2}$  to ensure stability of form.

**PROPOSITION 23.** If  $s^* = \frac{Q}{\beta} - G$  then the optimal results are affected by changes in  $\alpha$  as follows :

1.  $(G + s^*), a^*$  and  $\phi^*$  do not depend on  $\alpha$ .
2.  $\frac{\partial t^*}{\partial \alpha}$  has the same sign as  $(e - \frac{\alpha Q}{\beta})$ .  $\frac{\partial p^*}{\partial \alpha} > 0$ .
3.  $\frac{\partial z^*}{\partial \alpha} > 0$ ,  $\frac{\partial U^*}{\partial \alpha} > 0$ ,  $\frac{\partial \pi^*}{\partial \alpha} > 0$ , and  $\frac{\partial \psi^*}{\partial \alpha} > 0$ .

In this case, as  $\alpha$  increases, the form of the optimal compensation plan does not change, but the agent takes advantage of an increase in  $\alpha$  to increase the probability of a breakthrough. The system reward upon breakthrough and the shares of the firm and the agent remain the same, but both the firm and the agent have greater expected earnings due to the increase in  $p^*$ .

**PROPOSITION 24.** If  $s^* = \frac{Q}{\beta} - G$  then the optimal results are affected by changes in  $\beta$  as follows :

1.  $a^*$  does not depend on  $\beta$ .
2.  $\frac{\partial(G+s^*)}{\partial \beta}$ ,  $\frac{\partial t^*}{\partial \beta}$ ,  $\frac{\partial p^*}{\partial \beta}$ ,  $\frac{\partial z^*}{\partial \beta}$ ,  $\frac{\partial U^*}{\partial \beta}$ ,  $\frac{\partial \psi^*}{\partial \beta}$ , and  $\frac{\partial \phi^*}{\partial \beta}$  are all strictly negative.
3.  $\frac{\partial \pi^*}{\partial \beta} > 0$ .

Intuitively, an increase in  $\beta$  makes it easier for the naive firm to induce  $a^* = 0$  and therefore increases its profit even though the system expected benefit declines.

**PROPOSITION 25.** If  $s^* = \frac{Q}{\beta} - G$  then the optimal results are affected by changes in  $F$  and  $G$  as follows :

1.  $(G + s^*), a^*, t^*, p^*, z^*$  and  $U^*$  do not change as  $F$  or  $G$  varies.
  2.  $\pi^*, \psi^*$  and  $\phi^*$  depend on  $F$  and  $G$  through the sum  $(F + G)$ .
- $$\frac{\partial \pi^*}{\partial(F+G)} = \frac{\partial \psi^*}{\partial(F+G)} = p^* > 0 \text{ and } \frac{\partial \phi^*}{\partial(F+G)} < 0.$$

Here, an increase in  $(F + G)$  does not affect what the agent receives upon a breakthrough and hence does not affect the agent's actions or the probability of a breakthrough. The system receives a larger benefit upon a breakthrough and the entire increment is absorbed by the firm which reduces the agent's share.

**PROPOSITION 26.** If  $s^* = \frac{Q}{\beta} - G$  then the optimal results are affected by changes in  $Q$  as follows :

1.  $a^*$  does not depend on  $Q$ .
2.  $\frac{\partial(G+s^*)}{\partial Q}$ ,  $\frac{\partial t^*}{\partial Q}$ ,  $\frac{\partial p^*}{\partial Q}$ ,  $\frac{\partial z^*}{\partial Q}$ ,  $\frac{\partial U^*}{\partial Q}$ ,  $\frac{\partial \psi^*}{\partial Q}$ , and  $\frac{\partial \phi^*}{\partial Q}$  are all strictly positive.
3.  $\frac{\partial \pi^*}{\partial Q} < 0$ .

Intuitively, as  $Q$  increases, the naive firm has to increase  $s^*$  to dissuade the agent from selecting  $a^* > 0$ , which leads to a decrease in  $\pi^*$  even though the system expected benefit

increases. Clearly, the effect of an increase in  $Q$  is the opposite of that of an increase in  $\beta$  which motivates the agent to select a lower  $a$ .

**Case III.** In this case,  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$ , where  $\bar{s}$  is the (unique) solution of (47). This occurs if the following condition is satisfied :

$$\frac{\beta}{\alpha Q} < 1, \text{ (46) is satisfied, } (F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}, \text{ and } (F + G) \leq \frac{\alpha Q^2}{\beta^2}.$$

This may also happen if the following condition is satisfied :

$$\frac{\beta}{\alpha Q} < 1, (F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{1+Q}{\alpha Q} \right\}, \text{ and } (F + G) > \frac{\alpha Q^2}{\beta^2}.$$

In this case  $a^* > 0$ . Though we could not obtain  $s^*$  in a closed form here, the results presented in propositions 27 - 30 could be derived. These results are all obtained under the assumption that the form of the solution is not affected by infinitesimal variations in the parameters of the environment.

**PROPOSITION 27.** If  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$ , the optimal results are affected by changes in  $\alpha$  as follows :

$$1. \frac{\partial(G+s^*)}{\partial\alpha} < 0. \quad 2. \frac{\partial a^*}{\partial\alpha} > 0. \quad 3. \frac{\partial\pi^*}{\partial\alpha} > 0.$$

**PROPOSITION 28.** If  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$  the optimal results are affected by changes in  $\beta$  as follows :

$$1. \frac{\partial a^*}{\partial\beta} < 0. \quad 2. \frac{\partial\pi^*}{\partial\beta} > (=, <) 0 \text{ if } (F + G) > (=, <) \frac{1+Q}{\beta}.$$

Intuitively, an increase in  $\beta$  motivates the agent to select a lower level of difficulty. When  $(F + G) > \frac{1+Q}{\beta}$ , i.e.,  $a_S^* = 0$ , the increase in  $\beta$  enhances  $\pi_N^*$ , and the reverse happens if  $(F + G) < \frac{1+Q}{\beta}$  and  $a_S^* > 0$ .

**PROPOSITION 29.** If  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$ , changes in  $F$  and  $G$  affect the optimal results as follows :

1.  $F$  and  $G$  always affect the optimal results through the sum,  $(F + G)$ .

2.  $0 < \frac{\partial(G+s^*)}{\partial(F+G)} < 1$ , and  $\frac{\partial a^*}{\partial(F+G)} < 0$ .

3.  $\frac{\partial t^*}{\partial(F+G)}$ ,  $\frac{\partial p^*}{\partial(F+G)}$ ,  $\frac{\partial z^*}{\partial(F+G)}$ ,  $\frac{\partial\pi^*}{\partial(F+G)}$  and  $\frac{\partial\psi^*}{\partial(F+G)}$  are all strictly positive.

In addition,  $\frac{\partial\pi^*}{\partial(F+G)} = p^*$ , and  $\frac{\partial\psi^*}{\partial(F+G)} > p^*$ .

These results are similar to the ones listed in section 5.3 : an increase in fixed reward induces the system to increase the probability of breakthrough at the expense of quality.

**PROPOSITION 30.** If  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$ , the optimal results are affected by changes in  $Q$  as follows :

1. If  $(F + G) > -\frac{1}{\beta}$ , then  $\frac{\partial a^*}{\partial Q} > 0$ .
2.  $\frac{\partial \pi_N^*}{\partial Q}$  has the same sign as  $\left[ \frac{(1+Q)e}{\alpha Q} - \frac{(G+a^*)}{Q} e^{\beta(G+a^*)/Q} \right]$  and it can be positive or negative.

Since  $(F + G)$  is likely to be nonnegative, proposition 30 implies that an increase in  $Q$  will usually induce an increase in  $a^*$  for the naive firm. Regarding the effect of  $Q$  on  $\pi_N^*$ , intuitively it seems that when  $(F + G)$  is relatively large, an increase in  $Q$  will hinder the firm's ability to induce a lower  $a$  and will therefore hurt profits. On the other hand, if  $Q$  is large enough, the quality dependent reward is the dominant factor, and an increase in  $Q$  benefits the naive firm. It was not possible to analytically determine if there is a value of  $(F + G)$  where  $\frac{\partial \pi_N^*}{\partial Q}$  changes sign. However, the following two propositions provide conditions where  $a_N^* > 0$ , and  $\frac{\partial \pi_N^*}{\partial Q}$  have opposite signs.

**PROPOSITION 31.** If (46) is satisfied,  $\frac{\beta}{\alpha Q} < 1$  and  $(F + G) \leq \frac{Q}{\beta}$ , then  $a_N^* > 0$  and  $\frac{\partial \pi_N^*}{\partial Q} > 0$ .

**PROPOSITION 32.** If  $\frac{\beta}{\alpha Q} < 1$ ,  $(F + G) > \frac{1+Q}{\beta}$ ,  $(F + G) < \frac{\alpha Q^2}{\beta^2}$ ,  
 $(F + G) < \left\{ \frac{2Q}{\beta} + \frac{1}{\beta} - \frac{(1+Q)}{\alpha Q} \right\}$ , and  $\frac{\beta e}{\alpha Q^2} < 1$ , then  $a_N^* > 0$  and  $\frac{\partial \pi_N^*}{\partial Q} < 0$ .

**Numerical example.** Let us consider  $\alpha = 1$ ,  $\beta = .1$ , and  $Q = 1$ . It can be easily verified that the conditions of proposition 31 are satisfied if  $(F + G) \leq 10$ , and the conditions of proposition 32 satisfied if  $20 < (F + G) < 28$ . Table 3 presents  $\frac{\partial \pi_N^*}{\partial Q}$  for  $(F + G) = 11, \dots, 20$ .

Table 3 about here

It appears that if  $(F + G) \leq 13.5$ , then  $\frac{\partial \pi_N^*}{\partial Q} > 0$ , and if  $(F + G) \geq 14$  then  $\frac{\partial \pi_N^*}{\partial Q} < 0$ . A detailed numerical search shows that in this case, the sign changes at  $(F + G) \approx 13.745$ . Interestingly, for  $13.745 < (F + G) < 20$ ,  $a_N^* > 0$  but  $\frac{\partial \pi_N^*}{\partial Q} < 0$ .

**7.4. Comparison of the naive and the sophisticated firms : some selected topics.** In section 7.2 we made a general comparison of the optimal compensation plans for the naive and the sophisticated firm. We now address some specific issues.

**7.4.1. Comparison of  $a_N^*$  and  $a_S^*$ .** In the absence of a minimum expected utility constraint,  $a_N^*$  may be less than, equal to, or greater than  $a_S^*$ . We already discussed a case where  $a_S^* = 0$  but  $a_N^* > 0$  (Note 2 following proposition 22). The next proposition shows that when  $a_S^* > 0$ , it is still possible to have  $a_N^* > a_S^*$ .

**PROPOSITION 33.** Suppose  $\frac{\beta}{\alpha Q} < 1$ , (46) is satisfied, and  $(F + G) < \frac{1+Q}{\beta}$ . Then,

$$a_N^* < (=, >) a_S^* \iff \left( \frac{\beta e}{\alpha} \right) (1+Q) e^{-\beta(F+G)/(1+Q)} > (=, <) Q^2 \iff q^* > (=, <) 0,$$

where  $q^*$  corresponds to the optimal compensation plan of the sophisticated firm.

It follows from proposition 33 that, for any choice of  $\alpha$ ,  $\beta$ , and  $(F + G)$ , if  $Q$  is sufficiently large (which also ensures that  $\frac{\beta}{\alpha Q} < 1$  and (46) is satisfied), then  $a_N^* > a_S^* > 0$ . Intuitively, here the naive firm cannot effectively counter the impact of the large reputation factor.

**7.4.2. Cases where  $\pi_N^* = \pi_S^*$ .** The next three propositions list all the cases where  $\pi_N^* = \pi_S^*$ .

**PROPOSITION 34.**  $\pi_N^* = \pi_S^* = 0$  iff one of the following two conditions is satisfied:

(i)  $(F + G) \geq \frac{1+Q}{\beta}$  and  $(F + G) \leq \frac{1}{\alpha}$ , or (ii)  $(F + G) < \frac{1+Q}{\beta}$  and (18) is not satisfied.

**PROPOSITION 35.** If  $(F + G) \geq \frac{1+Q}{\beta}$ , then  $\pi_N^* = \pi_S^* > 0$  iff one of the following two conditions is satisfied : (i)  $\frac{\beta}{\alpha Q} \geq 1$  and  $(F + G) > \frac{1}{\alpha}$ , or (ii)  $\frac{\beta}{\alpha Q} < 1$  and  $(F + G) \geq \frac{\alpha Q^2}{\beta^2}$ .

In this case,  $a_N^* = a_S^* = 0$ . Clearly, if  $\alpha$ ,  $\beta$ , and  $Q$  remain the same and  $(F + G)$  exceeds a threshold, the conditions of proposition 35 are satisfied.

**PROPOSITION 36.** If  $(F + G) < \frac{1+Q}{\beta}$ , then  $\pi_N^* = \pi_S^* > 0$  iff the following condition is satisfied :

$$(56) \quad \left( \frac{\beta e}{\alpha} \right) (1 + Q) e^{-\beta(F+G)/(1+Q)} = Q^2.$$

Comparison with proposition 33 shows that  $a_S^* = a_N^* > 0$  if and only if (56) is satisfied (which also ensures that (18) is satisfied, i.e.,  $\pi_S^* > 0$ ). It follows from proposition 6 that if (56) is satisfied, then  $q_S^* = 0$ , i.e., the naive firm and the sophisticated firm have the same optimal compensation plan. For given values of  $\alpha$ ,  $\beta$ , and  $(F + G)$ , it can be shown that there is at most one value of  $Q$  which satisfies (56), i.e., unlike  $(F + G)$ , there is no threshold value of  $Q$  beyond which  $\pi_N^* = \pi_S^* > 0$  and  $a_N^* = a_S^* > 0$ . The following two examples show that for given values of  $\alpha$ ,  $\beta$ , and  $(F + G)$  it may or may not be possible to have a  $Q$  which satisfies (56).

**Example 1.** If  $(F + G) = 0$ ,  $\exists Q > 0$  which satisfies (56) and is given by

$$(57) \quad Q = \frac{1}{2} \left\{ \left( \frac{\beta e}{\alpha} + \sqrt{((\beta e)/\alpha)^2 + (4\beta e)/\alpha} \right) \right\}.$$

**Example 2.** If  $\alpha = 1$ ,  $\beta = 1$ , and  $(F + G) = 3$ , it is not possible to have a  $Q > 0$  which satisfies (56).

**7.4.3. The comparison of performance when  $(F+G)$  or  $Q$  is large.** The environment rewards the firm and the agent through  $(F+G)$  and  $Q$ . From proposition 35,  $a_N^* = a_S^* = 0$  and  $\pi_N^* = \pi_S^* > 0$  if  $(F+G)$  exceeds a threshold. In this case, the fixed reward from a breakthrough is too great to risk failure by selecting a higher level of difficulty than the minimum, 0, and even the naive firm finds it profitable to induce the agent to select  $a = 0$ . The next proposition compares the performance of the naive and the sophisticated firm when  $Q$  is large.

**PROPOSITION 37.** *If  $\alpha, \beta$  and  $(F+G)$  remain the same and  $Q \rightarrow \infty$ , then  $(G+s_N^*) \approx -\frac{Q}{\beta}$ , and the following hold:*

$$(58) \quad a_N^* \approx \frac{2}{\beta} \approx 2a_S^*,$$

$$(59) \quad t_N^* \approx \frac{1}{Q} \ln Q \approx 2t_S^*,$$

$$(60) \quad p_N^* \approx e^{-2} \approx p_S^* e^{-1},$$

$$(61) \quad \pi_N^* \approx \frac{Qe^{-2}}{\beta} \approx \pi_S^* e^{-1},$$

$$(62) \quad \psi_N^* \approx \frac{2Qe^{-2}}{\beta} \approx \psi_S^* \left(\frac{2}{e}\right),$$

$$(63) \quad U_N^* \approx \frac{Qe^{-2}}{\beta} \approx U_S^* \left(\frac{\alpha}{e}\right).$$

To obtain the intuition behind these results, we note from proposition 6 that when  $Q$  is large, the sophisticated firm uses  $(G+s^*) \approx 0$  and a negative  $q^*$ , i.e., it imposes a tax on the agent proportionate to quality. In contrast, the naive firm has to apply a negative fixed tax to extract a portion of the benefit the agent will obtain. This induces the agent to ensure that if there is a breakthrough, there is a substantial quality dependent reward to offset the tax, and hence a high level of difficulty is selected. This results in a lower probability of breakthrough compared to that of the sophisticated firm and leads to a smaller value of  $\pi^*$  and  $\psi^*$ . However,  $U_N^*$  may exceed  $U_S^*$  when  $Q$  is large.

It should be noted that  $Q \rightarrow \infty$  (for a fixed  $(F + G)$ ) means that the environment is sophisticated and that almost all the reward comes originally to the agent through the term  $Qa$ . Clearly, the sophisticated firm with its ability to relate compensation to quality can share this benefit more effectively than the naive firm and is substantially more profitable. However, this represents an extreme situation. In 7.4.4 we consider the special case,  $(F + G) = 0$ , and compare  $\pi_N^*$  and  $\pi_S^*$  when  $Q$  is moderately large.

**7.4.4. Relative performance when  $(F + G) = 0$ .** In this case the system does not derive any benefit just from a breakthrough. It follows from proposition 6 that :

1. The sophisticated firm can generate  $\pi > 0$  iff  $\frac{1+Q}{\beta} > \frac{\epsilon}{\alpha}$ .
2. The sophisticated firm will use  $s^* = -G$  and  $q^* = \sqrt{\frac{\beta\epsilon(1+Q)}{\alpha}} - Q$ , which induces  $a^* = \frac{1}{\beta} > 0$ .

It is interesting to note that here  $(G+s^*) = 0$  and  $(F-s^*) = 0$ , i.e., neither the sophisticated firm nor its agent receives any fixed reward. Also, the sophisticated firm always induces  $a_S^* = \frac{1}{\beta}$ .

Let us now consider the naive firm. It follows from propositions 17 and 18 that here the naive firm can obtain  $\pi > 0$  if and only if  $\frac{\beta}{\alpha Q} < 1$ , and the following condition holds:

$$(64) \quad Q e^{-Q/(1+Q)} > \frac{\beta}{\alpha}.$$

The LHS of (64) is a strictly increasing function of  $Q$  which approaches 0 when  $Q$  is small, and is unbounded above. Therefore, there is a threshold value of  $Q$  beyond which the naive firm can generate  $\pi > 0$ . Since  $(F + G) < \frac{\beta}{\alpha}$ , the optimal compensation plan will generate  $a^* > 0$ .

Let  $\bar{Q}$  denote the  $Q$  given by equation (57). As discussed in section 7.4.2, in this case  $\pi_N^* = \pi_S^* > 0$  if and only if  $Q = \bar{Q}$  (which also satisfies (64)). In comparing  $\pi_N^*$  and  $\pi_S^*$  for moderately large values of  $Q$ , it seems natural to use  $\bar{Q}$  as reference. We now present numerical results for the following two cases : (i)  $\alpha = 1$ ,  $\beta = .1$ , and (ii)  $\alpha = 1$  and  $\beta = .3$ . In both cases,  $\pi_S^* > 0$  for any  $Q \geq 0$ . In case (i),  $\pi_N^* > 0$  iff  $Q > .1104$ , and in case (ii),  $\pi_N^* > 0$  iff  $Q > .399$ . In each case,  $a_N^*$ ,  $\pi_N^*$ ,  $a_S^*$ ,  $\pi_S^*$  and the difference in profitability,  $(\pi_S^* - \pi_N^*)$ , were computed for  $Q = .1\bar{Q}, .2\bar{Q}, \dots, 1.5\bar{Q}, 2\bar{Q}, 5\bar{Q}$  and  $10\bar{Q}$ . Table 3 and Table 4 present results for case (i) and case (ii), respectively.

Table 4 about here

Table 5 about here

An inspection of the results show that for a reasonably large range around  $\bar{Q}$ ,  $\pi_N^*$  is not substantially inferior to  $\pi_S^*$ . A similar conclusion could be drawn from numerous other sets of parameter values. We thus feel justified in making the following statement : "When  $Q$  is moderately large, the naive firm performs reasonably well compared to the sophisticated firm."

**7.5. Summary.** In section 7 we have outlined the conditions under which the naive firm can generate  $\pi > 0$  and have compared the performances of the naive and the sophisticated firm. In particular, when  $(F + G)$  is large, both the naive and the sophisticated firm obtain the same results. When  $Q$  is large, it is possible to have a situation where the naive firm and the sophisticated firm both induce  $a^* > 0$  and obtain identical results. Also, at least in special cases, the naive firm performs reasonably well when  $Q$  is moderately large. However, when  $Q$  is very large, the results of the naive and the sophisticated firm diverge :  $t^*$  and  $a^*$  of the naive firm are both higher, and the probability of breakthrough, the firm's expected profit, and the expected system earnings are all lower. The agent's expected utility may, however, be greater.

The effect of changes in  $\alpha$  on the results of a naive firm, as far as we could determine analytically, are of the same nature as the effect of changes in  $\alpha$  on the results of the sophisticated firm. The same holds for the changes in  $(F + G)$ , and in particular,  $\frac{\partial \pi^*}{\partial (F+G)}$  is always equal to  $p^*$  for both the naive and the sophisticated firm (as long as the form of the solution does not change as  $(F + G)$  varies). However, the effect of changes in  $\beta$  and  $Q$  on the results of the naive firm have interesting points of departure.

An increase in  $\beta$  always decreases  $\pi_S^*$  if  $a_S^* > 0$ . In contrast,  $\frac{\partial \pi_N^*}{\partial \beta}$  may be positive or negative if  $a_N^* > 0$  and it can be positive when  $a_N^* = 0$ . Intuitively, even though an increase in  $\beta$  represents reduction in productivity resulting from a smaller probability of breakthrough for any level of difficulty above the lowest, it may make it easier for the naive firm to induce a lower level of difficulty and thereby enhance the probability of breakthrough.

We conclude this section with a review of the effect of  $Q$  on the performance of the naive firm. If  $(F + G) > 0$  and  $Q$  is very small, a change in  $Q$  has no influence on  $\pi_N^*$ . If  $Q$  is large,  $\pi_N^*$  will increase with  $Q$ . Between these two extremes, it is possible to have a range of values of  $Q$  where  $\pi_N^*$  is a strictly decreasing function of  $Q$ . Here, an increase in  $Q$  induces the agent to strive for higher quality. The naive firm cannot share this benefit efficiently, hence the conflict.

## 8. The Naive Firm With A Minimum Expected Utility Constraint.

**8.1. Introduction and the nature of the optimal compensation plan.** The naive firm with a minimum expected utility constraint solves the problem N2 defined in section 4.

Our presentation of this case will center on (a) the formulation of the optimal compensation plan, (b) comparative statics, and (c) the comparison of the results with those for the sophisticated firm with a minimum expected utility constraint, simply called the *sophisticated firm* throughout this section.

As in the case of a naive firm without a minimum expected utility constraint,  $\pi$  here is a continuous function of the firm's only decision variable  $s$ , which can always be chosen low enough to induce  $t = 0$ , i.e.,  $\pi_N^* \geq -m$  here. Also,  $a = 0$  if  $Q = 0$  and if  $Q > 0$ ,  $a = 0$  if  $s \geq (\frac{Q}{\beta} - G)$  and  $a > 0$  if  $s < (\frac{Q}{\beta} - G)$ . Propositions 38 and 39 parallel propositions 17 and 18 for the naive firm without a minimum expected utility constraint.

**PROPOSITION 38.** *If  $\frac{\beta}{\alpha Q} \geq 1$  then  $\pi_N^* > -m$  iff  $(F + G) > \frac{1}{\alpha}$ . If  $(F + G) > \frac{1}{\alpha}$  then  $s^* = F$ , and  $\pi^* > -m$  is given by (40).*

*If in addition  $(F + G) \geq \frac{1+Q}{\beta}$ , then  $\pi_N^* = \pi_S^*$ . Otherwise,  $\pi_N^* < \pi_S^*$ .*

**PROPOSITION 39.** *If  $\frac{\beta}{\alpha Q} < 1$ , the naive firm can generate  $\pi > -m$  only if condition (46) is satisfied, and if (46) is satisfied, then the naive firm can construct a compensation plan which induces  $a > 0$  and generates  $\pi > -m$ .*

We now restrict our attention to the case  $\frac{\beta}{\alpha Q} < 1$  and (46) is satisfied. Since  $\frac{\beta}{\alpha Q} < 1$ , in this case  $\frac{Q}{\beta} < \frac{1+Q}{\beta} - \frac{1}{\alpha Q}$  and the relation of  $\pi$  and  $s$  depends on whether  $(F + G) \leq \frac{Q}{\beta}$ ,  $\frac{Q}{\beta} < (F + G) < (\frac{1+Q}{\beta} - \frac{1}{\alpha Q})$ , or  $(F + G) \geq (\frac{1+Q}{\beta} - \frac{1}{\alpha Q})$  as follows.

1. If  $(F + G) \geq [\frac{1+Q}{\beta} - \frac{1}{\alpha Q}]$  then  $\pi$  for  $s > s_1$  is a unimodal function which is uniquely maximized by  $(F, 0)$ . Here,  $s^* = F > (\frac{Q}{\beta} - G)$  and  $a^* = 0$ .
2. If  $(F + G) \leq \frac{Q}{\beta}$ , then  $\pi$  is a unimodal function of  $s$  for  $s > s_1$  with a unique maximum at  $\bar{s}$  which is uniquely given by

$$(65) \quad \frac{e}{\alpha} = e^{\beta(G+\bar{s})/Q} \{(G + \bar{s})(1 + Q) - Q(F + G)\}.$$

Here  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$  and  $a^* > 0$ . In the remainder of section 8 we always define  $\bar{s}$  by equation (65).

3. If  $\frac{Q}{\beta} < (F + G) < \frac{1+Q}{\beta} - \frac{1}{\alpha Q}$  then  $\pi$  is a bimodal function of  $s$  with the modes at (i)  $\bar{s} < (\frac{Q}{\beta} - G)$  which is the unique solution of (65), and (ii)  $F > (\frac{Q}{\beta} - G)$ . It can be shown that there is a value of  $(F + G)$  where both these plans generate the same  $\pi$ . If  $(F + G)$  exceeds this threshold,  $(F, 0)$  is strictly optimal (i.e.,  $a^* = 0$ ) while if  $(F + G)$  is below this threshold,  $(\bar{s}, 0)$  is strictly optimal (i.e.,  $a^* > 0$ ).

The next proposition formally states the optimal compensation plan.

- PROPOSITION 40.** If  $\frac{\beta}{\alpha Q} < 1$  and (46) is satisfied, then  $\exists \gamma \in (\frac{Q}{\beta}, \frac{1+Q}{\beta} - \frac{1}{\alpha Q})$  such that  
(i) if  $(F + G) < \gamma$ , the optimal compensation plan is  $(\bar{s}, 0)$ ,  $\bar{s} < (\frac{Q}{\beta} - G)$ , and it induces  $a^* > 0$ ,  
(ii) if  $(F + G) > \gamma$ , the optimal compensation plan is  $(F, 0)$ ,  $F > (\frac{Q}{\beta} - G)$ , and it induces  $a^* = 0$ , and  
(iii) if  $(F + G) = \gamma$ ,  $\pi$  is maximized by both  $(\bar{s}, 0)$  and  $(F, 0)$  which generate the same expected profit.

**8.2. Results and comparative statics.** We will only consider the case where  $\pi^* > -m$ .

**Case 1.**  $a^* = 0$ . In this case, the optimal compensation plan for the naive firm is  $(F, 0)$ , and the expressions for  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  are identical to those of the sophisticated firm with a minimum expected utility requirement when  $(F + G) \geq \frac{1+Q}{\beta}$  and hence the results of comparative statics are identical as well. If  $(F + G) \geq \frac{1+Q}{\beta}$ , then  $\pi_N^* = \pi_S^*$ . Otherwise, i.e., if  $\gamma < (F + G) < \frac{1+Q}{\beta}$ , then  $\pi_N^* < \pi_S^*$ .

**Case 2.**  $a^* > 0$ . Here  $s^* = \bar{s} < (\frac{Q}{\beta} - G)$ . We could not obtain  $\bar{s}$  and hence any of  $a^*$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  in a closed form. However, the following results regarding  $a^*$  and  $\pi^*$  could be derived.

**PROPOSITION 41.** If  $a_N^* > 0$ , then  $a_N^* < a_S^*$  and  $\pi_N^* < \pi_S^*$ .

Therefore, unlike the case when  $a_N^* = 0$ , it is never possible to have  $\pi_N^* = \pi_S^*$  when  $a_N^* > 0$ . Only the following results could be obtained from the comparative statics.

**PROPOSITION 42.** If  $a^* > 0$ ,  $\pi^* > 0$ , and the form of the solution is not affected by infinitesimal variations in parameter values, then the optimal results depend on parameters of the environment as follows.

1. **Effect of  $\alpha$ .**  $\frac{\partial(G+s^*)}{\partial\alpha} < 0$  and  $\frac{\partial a^*}{\partial\alpha} > 0$ .
2. **Effect of  $\beta$ .**  $\frac{\partial(G+s^*)}{\partial\beta} < 0$  and  $\frac{\partial a^*}{\partial\beta} < 0$ .
3. **Effect of  $F$  and  $G$ .**  $F$  and  $G$  enter the expressions for  $(G+s^*)$ ,  $a^*$ ,  $t^*$ ,  $\pi^*$  and  $\psi^*$  through the sum  $(F+G)$ .  $(G+s^*)$ ,  $t^*$ ,  $p^*$ ,  $\pi^*$  and  $\psi^*$  are strictly increasing and  $a^*$  is strictly decreasing in  $(F+G)$ . Also,  $\frac{\partial(G+s^*)}{\partial(F+G)} < \frac{1}{1+Q} < 1$ ,  $\frac{\partial\pi^*}{\partial(F+G)} = p^*$ , and  $p^* < \frac{\partial\psi^*}{\partial(F+G)} < 1$ .
4. **Effect of  $Q$ .**  $\frac{\partial t^*}{\partial Q} > 0$ ,  $\frac{\partial\pi^*}{\partial Q} > 0$ , and  $\frac{\partial\psi^*}{\partial Q} > 0$ . Also, if  $(F+G) \geq 0$ , then  $\frac{\partial a^*}{\partial Q} > 0$ .

These results are generally similar to those for the sophisticated firm except for the following : (i) an increase in  $\alpha$  does not affect  $a_S^*$  but it increases  $a_N^*$ , and (ii) if  $(F+G) = 0$  then  $\frac{\partial\pi_N^*}{\partial Q} > 0$  but  $\frac{\partial\pi_S^*}{\partial Q} = 0$ .

**8.3. The naive and the sophisticated firm : a comparison of results.** It follows from propositions 14, 40 and 41 that if  $(F+G) \geq \frac{1+Q}{\beta}$ , then  $a_N^* = a_S^* = 0$ . Otherwise,

$a_N^* < a_S^*$ . With regard to expected profits, we find that it is possible to have  $\pi_N^* = \pi_S^*$  only in the following two cases:

(i) if  $\pi_N^* = \pi_S^* = 0$ , or (ii)  $(F + G) \geq \frac{1+Q}{\beta}$  and  $(F + G) > \frac{1}{\alpha}$ , i.e.,  $a_N^* = a_S^* = 0$ .

It is never possible to have identical results for the naive and the sophisticated firm when  $a_S^* > 0$ . This arises from the fact that unlike the case without a minimum expected utility constraint, here  $q_S^* = 1$ . Consequently, it is possible to have identical results for the naive and the sophisticated firm only if  $a^* = 0$ , i.e., the difference in  $q$  does not play a role.

When  $(F + G)$  exceeds  $\frac{1}{\alpha}$  and  $\frac{1+Q}{\beta}$ , the naive firm and the sophisticated firm obtain identical results. The following proposition compares the performance of the naive and the sophisticated firm when  $Q$  is large.

**PROPOSITION 43.** *If  $\alpha, \beta, F$  and  $G$  are held fixed and  $Q \rightarrow \infty$ , then  $s^* = \bar{s} \approx F$ , and the following hold.*

$$(66) \quad a_N^* \approx \frac{1}{\beta} \approx a_S^*,$$

$$(67) \quad t_N^* \approx \frac{1}{\alpha} \ln Q \approx t_S^*,$$

$$(68) \quad p_N^* \approx \frac{1}{e} \approx p_S^*,$$

$$(69) \quad \pi_N^* \approx \frac{Q e^{-1}}{\beta} \approx \pi_S^*,$$

and

$$(70) \quad \psi_N^* = \pi_N^* + m + t_N^* \approx \psi_S^*.$$

It is interesting to note that when  $Q$  is large i.e., the external reputation dominates the solution, the inability of the naive firm to relate compensation to quality results in lower values of both  $a^*$  and  $\pi^*$ . However, when  $Q$  is very large, this deficiency has very little impact and the results of the naive and the sophisticated firm converge. In order to compare  $\pi_N^*$  and  $\pi_S^*$  for moderately large values of  $Q$ , we once again consider  $(F + G) = 0$  and present  $\pi_N^*$  and  $\pi_S^*$  for the values of  $\alpha, \beta$ , and  $Q$  used in tables 3 and 4, and  $m = 0$ . Clearly, here also we can make the claim, “When  $Q$  is moderately large, the naive firm performs reasonably well compared to the sophisticated firm.”

Table 6 about here

Table 7 about here

**8.4. Summary.** In general, the results of this section are similar to those of a naive firm without a minimum expected utility constraint. There is however an important distinction. In the absence of a minimum expected utility constraint, the naive firm finds it difficult to share the benefits of external reputation, and this is significant when  $Q$  is large. This results in a divergence in the results of the naive and the sophisticated firm. In contrast, if a minimum expected utility constraint is present, the naive firm can perform almost as well as the sophisticated firm when  $Q$  is large.

## 9. Conclusion

**9.1 Summary.** We have developed a model which incorporates the level of difficulty  $a$  an agent selects as well as the amount of effort  $t$  he devotes to a project. If the project is completed successfully, the system consisting of the firm and the agent receives the fixed reward  $(F + G)$  and the quality dependent reward  $(1 + Q)a$ . Clearly, a more difficult project, if completed, generates a greater reward for the system. We obtain the result that the firm will induce the agent to undertake the project only if  $(F + G)$  or  $Q$  is large enough, i.e., the system will receive an attractive reward upon a breakthrough.

The optimal compensation plan of the firm depends on how much control it has over the agent's actions. The firm's control is determined by two factors, (i) whether the firm is sophisticated or naive, and (ii) whether there exists a minimum expected utility  $m$ , known to the firm, such that the agent will accept employment if and only if his expected utility is at least  $m$ .

The sophisticated firm can relate the agent's compensation to the quality of the output. The naive firm cannot do so and can only offer a fixed reward for a breakthrough. If the agent requires a minimum expected utility  $m$  to accept employment and the firm knows  $m$ , the firm can always adjust a base income, paid regardless of whether a breakthrough occurs, to ensure that the agent's expected utility never exceeds  $m$ . This ability to use the base income to its advantage provides the firm with an additional measure of control over the agent. If the firm does not have this ability, we assume that the base income is zero, and this assumption has no impact on the results.

We analyze the following four cases separately, (i) the sophisticated firm without a minimum expected utility constraint, (ii) the sophisticated firm with a minimum expected utility constraint, (iii) the naive firm without a minimum expected utility constraint, and (iv) the naive firm with a minimum expected utility constraint. The sophisticated firm with a minimum expected utility constraint has the largest degree of control over the agent and

can obtain first best results, i.e., results it would obtain if it could directly observe  $t$  and  $a$ . The naive firm without a minimum expected utility constraint has the least control over the agent. In spite of the variation in the firm's control over the agent, the results from the four cases are largely similar. For example, an increase in  $(F + G)$  always increases the expected profit of the firm.

The primary objective of the paper is to determine how rewards derived from the environment affect the performance of the system, in particular the expected profit of the firm and the level of project difficulty induced. The terms  $(F + G)$  and  $Q$  have countervailing influences on the choice of level of difficulty. As  $(F + G)$  increases, the firm attempts to dissuade the agent from selecting a high level of difficulty which will reduce the chance of success. If  $(F + G)$  is very large, the optimal plans in all the four cases induce the lowest level of difficulty ( $a = 0$ ). For example, the sophisticated firm without a minimum expected utility constraint may use a disincentive for quality (i.e.,  $q < 0$ ) to align the agent's objective with its own. The naive firm, able to only use a fixed reward, makes it large in order to make the agent refrain from trying difficult projects.

It is interesting to note that when  $(F + G)$  is large, the results indicate that the sophisticated firm may use its ability to relate compensation to quality to punish the agent for trying to attempt a difficult project. For example, we sometimes hear allegations that persons in charge of defense projects tend to discourage subordinates from testing the quality of the output too closely. The closed nature of the system, i.e., the lack of a reputation factor for the agent, the fact that information about the output is largely classified, and the presence of a substantial fixed reward jointly provide the incentive to stress completion rather than quality. It therefore seems prudent to allow public scrutiny of completed projects unless security concerns prevail.

As  $Q$  becomes large, the potential benefit from quality motivates the firm to induce the agent to attempt difficult projects. For example, the sophisticated firm without a minimum expected utility constraint may offer the agent an incentive for quality ( $q > 0$ ). When  $Q$  is very large, the firm does not need to provide any incentive of its own for quality and imposes a "quality tax" on the agent to share the benefits.

Of the four cases we consider, the naive firm without a minimum expected utility constraint has the least control over the agent. One consequence of this lack of control is, when  $(F + G)$  and  $Q$  are both moderately large, the firm may like to reduce quality and thus enhance the chance of success, but be unable to do so due to the effect of reputation on the agent. In such a situation, an increase in  $Q$  may motivate the agent to choose an even more difficult project and thereby reduce the expected profit of the firm. An increase in  $\beta$ , which reduces the chance of breakthrough if the agent chooses above the lowest level of difficulty, may now benefit the firm since it steers the agent away from selecting a high level of difficulty. In all

the other three cases, the greater degree of control of the firm on the actions of the agent helps avoid this conflict and an increase in  $Q$  always benefits the firm as long as the firm chooses to induce  $a > 0$ .

The naive firm without a minimum expected utility constraint is also characterized by the fact that when  $Q$  is large, the level of project difficulty is greater than in all the other three cases. Even though this is accompanied by a smaller chance of breakthrough, society at large may benefit from it.

In general, we find that when  $(F + G)$  is large, the naive firm and the sophisticated firm induce the lowest level of difficulty and obtain identical results. Also, when  $Q$  is large, the naive and the sophisticated firm may obtain close and even identical results. Here, the reputation factor serves to provide the motivation for quality which the naive firm cannot, and this reduces the gap in performance.

The results underscore the need to have a sophisticated environment which can reward quality and erase the effect of a naive firm which cannot. The inability of the environment to reward quality, on the other hand, ensures mediocrity rather than excellence.

**9.2. Directions For Future Research.** The model developed attempts to capture the essence of an incentive problem where quality is involved and in so doing makes several simplifications. Relaxing any of the simplifying assumptions will be a direction for future research. Some components of the model where an extension might be interesting are listed below.

**1. The ability of the firm to evaluate quality.** The model presented assumes that the firm is either sophisticated or naive, i.e., it can relate the agent's reward to a perfect measure of  $a$ , or cannot relate reward to  $a$  at all. A natural extension will be to consider a firm which can base the compensation on an imperfect measure of quality, i.e., with uncertainty and possibly a bias. This extension is meaningful only if the agent is assumed to be risk averse since otherwise the expected utility of the agent will once again be a linear function of  $a$  similar to  $(s + qa)$  and the results similar to those for the sophisticated firm.

**2. The quality obtained from a breakthrough.** The model developed assumes that if a breakthrough is achieved, the level of difficulty is equal to the quality obtained. A relatively simple extension to this will be to assume that the quality achieved is an increasing function of  $a$ ,  $A(a)$ , rather than  $a$  itself. If  $A(a)$  is assumed to have an inverse such that  $p_2(a)$  can be expressed as a function of  $A$ , the model presented in the paper can be easily reformulated in terms of  $A$ .

A more general extension will be to assume that if the agent makes a breakthrough, the quality attained is a random variable with a distribution which depends on the level of difficulty  $a$ . If the agent is risk neutral, he will only be affected by the expected value of the

uncertain quality, and this extension will reduce to the one where quality is a deterministic function of  $a$ . Therefore, we will obtain additional insight only if the agent is assumed to be risk averse.

**3. Compatibility of the rewards of the firm and the agent.** A key assumption of the model is that the rewards which the firm and the agent receive from the environment upon a breakthrough,  $(F + a)$  for the firm and  $(G + Qa)$  for the agent, are linearly related, and  $Q \geq 0$ . A relatively simple extension will be to introduce nonlinearity here. A more interesting extension will be to allow the rewards of the firm and the agent to be partially incompatible. For example, an advertising agency may develop a campaign which earns the agency reputation for creativity, but is unrelated or even detrimental to the client organization. The ability of the firm to dissuade the agent from engaging in such dysfunctional behavior will determine profitability.

**4. The probability of breakthrough.** A relatively simple extension will be to retain the multiplicative form of the probability of breakthrough,  $p(t, a) = p_1(t)p_2(a)$  used here, but make  $p_1(t)$  and  $p_2(a)$  more general functions of  $t$  and  $a$ . A more interesting possibility is now presented.

The assumption that  $p(t, a) = p_1(t)p_2(a)$  is similar to assuming that the agent makes several attempts to complete the project (the number of trials is proportional to  $t$ ), and selects the same level of difficulty for each trial. However, it is conceivable that an initial failure may lead an agent to lower his goals in subsequent trials. This may generate a  $p(t, a)$  which is not multiplicatively separable in  $t$  and  $a$ . An alternative approach is to divide the period into multiple subperiods to allow the agent to make adjustments in effort and level of difficulty during a period.

**5. The number of projects.** In this paper, the agent is assumed to undertake a single project. A natural extension will be to assume that the agent selects a portfolio of projects rather than a single one. This extension will be particularly relevant to the study of a salesforce which is trying to make sales to several accounts, and of an advertising agency which is developing multiple advertising campaigns.

It is possible to conceive of other generalizations of the model. For example, it will be interesting to extend it to multiple periods where the past achievements of an agent may affect the recognition he will get for work done in the later periods. We hope that a thorough analysis of the problem described and its extensions will contribute to a better understanding of organizations and agents when quality is a factor.

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**Table 1**

Summary of Comparative Statics Results for the Sophisticated Firm

**Case 1.  $a^* = 0$**

Effect of increased

Effect on	$\alpha$	$\beta$	$Q$	$(F + G)$
$(G + s^*)^1$	↓	0	0	↑
$t^*$	↑	0	0	↑
$p^*$	↑	0	0	↑
$z^{*1}$	↑	0	0	↑
$U^{*1}$	↑	0	0	↑
$\pi^*$	↑	0	0	↑
$\psi^*$	↑	0	0	↑
$\phi^{*1}$	↓	0	0	↓

**Case 2.  $a^* > 0$**

Effect of increased

Effect on	$\alpha$	$\beta$	$Q$	$(F + G)$
$(G + s^*)^1$	↓	↑	↓	↑
$(Q + q^*)^1$	↓	↑	↑	↓
$t^*$	↑	↓	↑	↑
$a^*$	0	↓	↑	↓
$p^*$	↑	↑	↑	↑
$z^{*1}$	↑	↓	↑	↑
$U^{*1}$	↑	↓	↑	↑
$\pi^*$	↑	↓	↑	↑
$\psi^*$	↑	↓	↑	↑
$\phi^{*1}$	↓	↑	↓	↓

0 no change,      ↑ goes up,      ↓ goes down,      ↑ may go up or down.

1 → relevant only for the sophisticated firm without a minimum expected utility constraint.

**Table 2**

$(F + G)$	$\pi(s^\dagger)$	$\pi(\bar{s})$	$\pi^*$	$a^*$	$t^*$	$p^*$
1.60	0.07018	0.08152	0.08152	0.1462	0.1647	0.1432
1.65	0.08095	0.08885	0.08885	0.1282	0.1719	0.1500
1.70	0.09232	0.09653	0.09653	0.1100	0.1791	0.1570
1.75	0.10425	0.10455	0.10455	0.0918	0.1864	0.1639
1.80	0.11672	0.11292	0.11672	0.0	0.2939	0.2546
1.85	0.12971	0.12165	0.12971	0.0	0.3076	0.2648
1.90	0.14319	0.13074	0.14319	0.0	0.3209	0.2745
1.95	0.15715	0.14019	0.15715	0.0	0.3339	0.2839

**Table 3**

$(F + G)$	$\frac{\partial \pi_N^*}{\partial Q}$	$(F + G)$	$\frac{\partial \pi_N^*}{\partial Q}$
10.5	0.6938	15.5	-0.4821
11.0	0.6022	16.0	-0.6342
11.5	0.5050	16.5	-0.7944
12.0	0.4025	17.0	-0.9604
12.5	0.2942	17.5	-1.1358
13.0	0.1798	18.0	-1.3189
13.5	0.0606	18.5	-1.5116
14.0	-0.0663	19.0	-1.7109
14.5	-0.1979	19.5	-1.9197
15.0	-0.3357	20.0	-2.1381

Table 4

 $\alpha = 1, \beta = .1, \bar{Q} = .6747.$ 

	$Q$	$a_N^*$	$\pi_N^*$	$a_s^*$	$\pi_s^*$	$(\pi_s^* - \pi_N^*)$
1	0.0675		0.0000	10.0000	0.9637	0.9637
2	0.1349	2.0721	0.0720	10.0000	1.0885	1.0166
3	0.2024	4.1796	0.4933	10.0000	1.2170	0.7237
4	0.2699	5.6233	0.8855	10.0000	1.3488	0.4633
5	0.3374	6.7188	1.2028	10.0000	1.4837	0.2809
6	0.4048	7.5998	1.4625	10.0000	1.6214	0.1589
7	0.4723	8.3351	1.6818	10.0000	1.7617	0.0799
8	0.5398	8.9648	1.8723	10.0000	1.9045	0.0321
9	0.6072	9.5140	2.0422	10.0000	2.0495	0.0073
10	0.6747	10.0000	2.1967	10.0000	2.1967	0.0000
11	0.7422	10.4348	2.3396	10.0000	2.3459	0.0062
12	0.8097	10.8272	2.4737	10.0000	2.4970	0.0233
13	0.8771	11.1841	2.6009	10.0000	2.6499	0.0490
14	0.9446	11.5106	2.7225	10.0000	2.8045	0.0819
15	1.0121	11.8109	2.8397	10.0000	2.9607	0.1209
16	1.3494	13.0192	3.3828	10.0000	3.7632	0.3804
17	3.3735	16.2136	6.2450	10.0000	9.0671	2.8221
18	6.7471	17.8279	10.8330	10.0000	18.8229	7.9899

Table 5

$$\alpha = 1, \beta = .3, \bar{Q} = 1.3986.$$

	$Q$	$a_N^*$	$\pi_N^*$	$a_S^*$	$\pi_S^*$	$(\pi_S^* - \pi_N^*)$
1	0.1399		0.0000	3.3333	0.0332	0.0332
2	0.2797		0.0000	3.3333	0.0639	0.0639
3	0.4196	1.0514	0.0014	3.3333	0.1020	0.1006
4	0.5594	1.6216	0.0521	3.3333	0.1466	0.0944
5	0.6993	2.0556	0.1287	3.3333	0.1967	0.0680
6	0.8391	2.4034	0.2090	3.3333	0.2518	0.0428
7	0.9790	2.6918	0.2879	3.3333	0.3112	0.0232
8	1.1189	2.9366	0.3645	3.3333	0.3744	0.0099
9	1.2587	3.1481	0.4389	3.3333	0.4412	0.0024
10	1.3986	3.3333	0.5112	3.3333	0.5112	0.0000
11	1.5384	3.4974	0.5820	3.3333	0.5842	0.0022
12	1.6783	3.6441	0.6514	3.3333	0.6598	0.0083
13	1.8181	3.7763	0.7198	3.3333	0.7378	0.0181
14	1.9580	3.8961	0.7872	3.3333	0.8182	0.0310
15	2.0979	4.0054	0.8540	3.3333	0.9007	0.0467
16	2.7971	4.4360	1.1808	3.3333	1.3406	0.1598
17	6.9928	5.5041	3.0801	3.3333	4.5399	1.4598
18	13.9857	6.0103	6.2280	3.3333	10.8029	4.5749

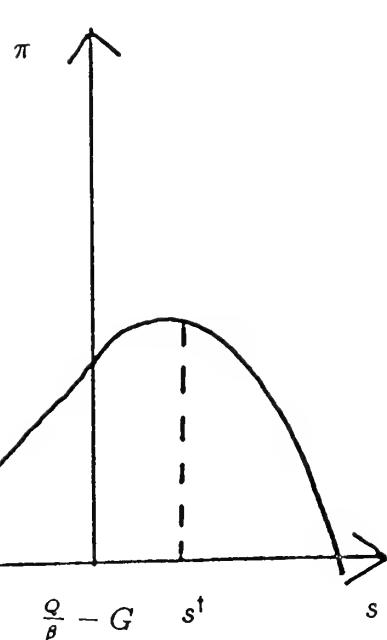
Table 6,  $\alpha = 1, \beta = .1$ 

	$Q$	$a_N^*$	$\pi_N^*$	$a_S^*$	$\pi_S^*$	$(\pi_S^* - \pi_N^*)$
1	0.0675		0.0000	10.0000	1.5591	1.5591
2	0.1349	2.0138	0.0766	10.0000	1.7461	1.6694
3	0.2024	3.9196	0.5431	10.0000	1.9365	1.3934
4	0.2699	5.1276	1.0066	10.0000	2.1301	1.1236
5	0.3374	5.9724	1.4093	10.0000	2.3266	0.9173
6	0.4048	6.5985	1.7636	10.0000	2.5256	0.7620
7	0.4723	7.0806	2.0836	10.0000	2.7269	0.6433
8	0.5398	7.4624	2.3796	10.0000	2.9303	0.5507
9	0.6072	7.7713	2.6586	10.0000	3.1356	0.4770
10	0.6747	8.0254	2.9252	10.0000	3.3427	0.4175
11	0.7422	8.2374	3.1827	10.0000	3.5514	0.3687
12	0.8097	8.4165	3.4336	10.0000	3.7616	0.3281
13	0.8771	8.5691	3.6793	10.0000	3.9732	0.2939
14	0.9446	8.7005	3.9212	10.0000	4.1861	0.2649
15	1.0121	8.8144	4.1601	10.0000	4.4002	0.2401
16	1.3494	9.2079	5.3298	10.0000	5.4863	0.1565
17	3.3735	9.8191	12.2771	10.0000	12.3112	0.0341
18	6.7471	9.9483	24.1405	10.0000	24.1501	0.0096

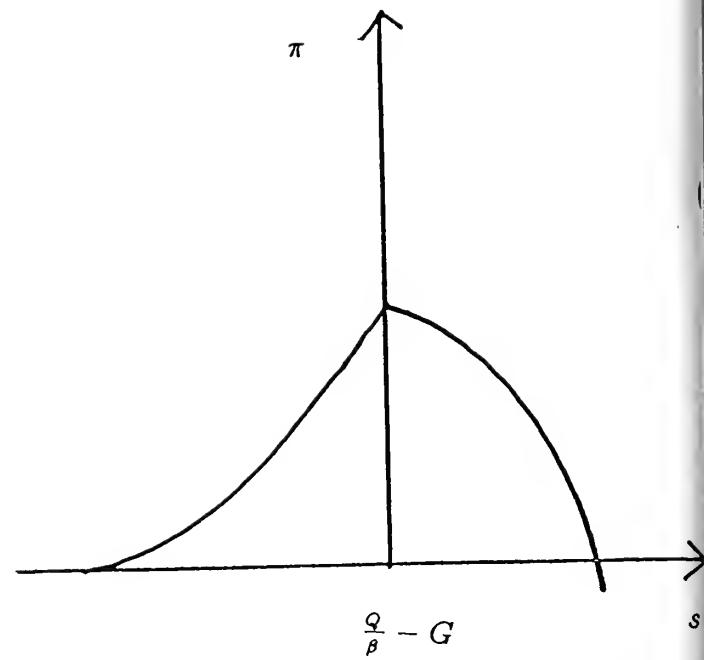
Table 7,  $\alpha = 1, \beta = .3$ 

	$Q$	$a_N^*$	$\pi_N^*$	$a_S^*$	$\pi_S^*$	$(\pi_S^* - \pi_N^*)$
1	0.1399		0.0000	3.3333	0.0629	0.0629
2	0.2797	0.0000	0.0000	3.3333	0.1187	0.1187
3	0.4196	1.0398	0.0016	3.3333	0.1864	0.1848
4	0.5594	1.5231	0.0641	3.3333	0.2640	0.1999
5	0.6993	1.8621	0.1648	3.3333	0.3496	0.1848
6	0.8391	2.1122	0.2776	3.3333	0.4420	0.1644
7	0.9790	2.3033	0.3955	3.3333	0.5402	0.1447
8	1.1189	2.4529	0.5160	3.3333	0.6434	0.1274
9	1.2587	2.5724	0.6385	3.3333	0.7510	0.1125
10	1.3986	2.6694	0.7626	3.3333	0.8624	0.0999
11	1.5384	2.7492	0.8882	3.3333	0.9773	0.0891
12	1.6783	2.8156	1.0153	3.3333	1.0951	0.0799
13	1.8181	2.8715	1.1438	3.3333	1.2157	0.0719
14	1.9580	2.9189	1.2737	3.3333	1.3388	0.0651
15	2.0979	2.9594	1.4049	3.3333	1.4641	0.0592
16	2.7971	3.0951	2.0793	3.3333	2.1181	0.0388
17	6.9928	3.2854	6.5105	3.3333	6.5188	0.0083
18	13.9857	3.3204	14.4631	3.3333	14.4654	0.0023

$$(A) \quad s^* = s^\dagger$$

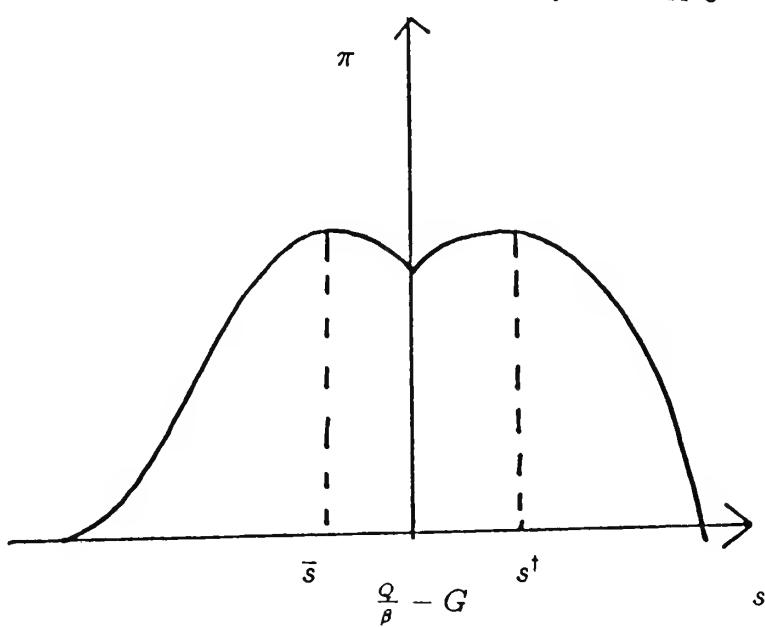


$$(B) \quad s^* = \frac{Q}{\beta} - G$$



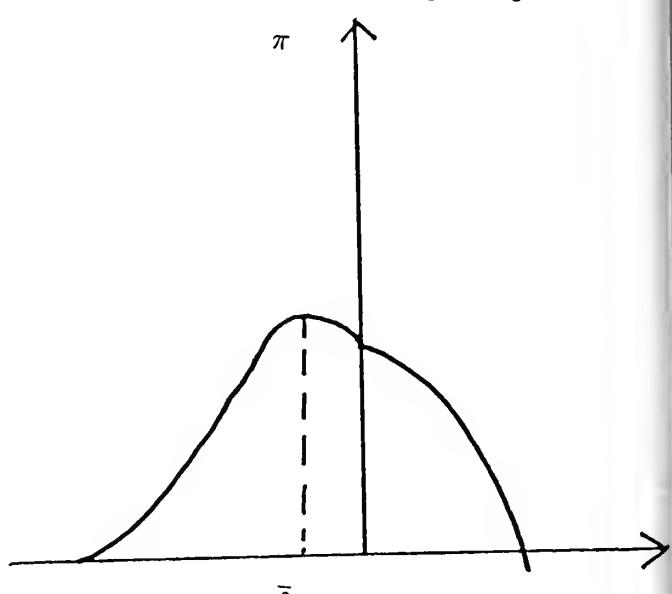
(C)

$$s^* \text{ may be } s^\dagger \text{ or } \bar{s}$$



(D)

$$s^* = \bar{s}$$



**Figure 1**

$\pi$  as a function of  $s$  for the naive firm without a minimum expected utility constraint.





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